Nonlinear Pricing with Local Network Effects*

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Abstract

I present a model of second-degree price discrimination of a local network good where consumers are located in a social network, and their private valuations are endogenous and depend on the adoption decisions of their connections. A monopolistic seller decides the lowest contract available for purchase which determines participation through a threshold game of complements played among the buyers. Relative to a benchmark with consumers' valuations being exogenous, participation is likely to be higher with local network externalities since a participating consumer generates positive externalities to other participating consumers through direct and indirect social relationships. Externalities are broken down into a "market size effect" (higher participation) and a "distribution effect" (participating consumers derive increased value from consumption). The importance of these externalities depends on identifiable properties of the underlying network structure. Under certain network structures, the monopolist has no choice but to serve unprofitable consumers, as exclusion of low-demand types can lead potential high-demand consumers to leave the market or become unprofitable. A situation where the monopolist decides not to enter the market, despite production being socially desirable, may occur.

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1 Introduction

For certain products, our purchasing decisions are influenced by the decisions of our social relations. For instance, the phone contract you choose to subscribe to depends on how many in your social network you can reach, and whether you should join a certain file-sharing service may depend on whether your friends are on the same service. In other words, buyers generate positive externalities to other buyers they have relations with.

This paper addresses the strategy of a monopolistic seller of such a "local network good". When buyers generate externalities to other buyers with whom they have relations, and these externalities are partly extracted by the seller, how are profits maximized? The specific situation addressed in this paper is a monopolistic firm offering a menu of vertically differentiated products. Because externalities are generated between connected consumers, the set of profit maximizing contracts ultimately depends on how different types of consumers are connected. Key properties of the network structure determining the set of profit maximizing contracts are identified in this paper.

Products exhibiting local network externalities range from fashion to social networking sites to telephony. The subset of products considered in the present paper are products where it is feasible to imperfectly discriminate across consumers by offering a menu of products of different quantity or quality as first analyzed by Mussa and Rosen (1978) and Maskin and Riley (1984). The key addition to the standard model is that the valuation of a consumer (its privately observed "type") is endogenously determined by the participation of its connections. Thus, when a consumer experiences an increase in the number of friends buying a compatible product, then this consumer will have a higher benefit of a given quantity (or quality) and is inclined to upgrade to a contract with higher quantity whenever contracts are incentive compatible.

There are various examples of products exhibiting local network effects which also are offered as menus of vertically differentiated products: (i) In addition to online storage, users of cloud services like Dropbox and Google Drive may benefit from sharing files with friends and colleagues using the same service. The more individuals your account is connected to, the higher is the benefit of an extra byte of storage, and you may be more likely to go for a contract with more storage. (ii) Mobile operators typically offer a set of plans with a given amount of SMS, talk time and data included in the monthly fee. To some extent, the optimal plan for a consumer depends on the number of individuals in his social network using a compatible service. For "friends and family plans" the benefit is limited to the number of friends being customers at the same mobile operator. (iii) Until recently a premium account for USD 9.99/month was needed for accessing group video calls on Skype.[1] Those with many Skype contacts would therefore be more inclined to accept the premium offer.

In these markets, participation of a consumer affects the benefit of other participating consumers. Therefore, the seller is prone to offer contracts to seemingly unprofitable and scarcely

[1] https://web.archive.org/web/20140330011125/http://www.skype.com/en/premium/ . On April 28, 2014, group video calls became free of charge.

connected consumers. The purchasing decision of one consumer depends on the decisions taken by those he is connected to. In effect, the consumers are playing a *threshold game of complements*, a game where a consumer buys only if at least a given number of his friends buys the same product. At a given threshold, participation in this game crucially depends on the network structure. Thus, the strength of local externalities are determined by *how* consumers are linked. For instance, one consumer's purchasing decision may depend on his friend buying, who again buys only if another friend buys. Thus, the decision of your friends' friends' (...) friends' friends may be detrimental for your own purchasing decision. In that case the impact from local externalities is strong, and the seller may want to sell to as many as possible. If decisions of indirect connections are less important for your decision, then this impact is weaker, and the seller may find it optimal to offer a set of products which only highly connected (high-demand) consumers would be willing to buy.

The network externalities consumers generate by participating can be separated into two types of effects. First, the *market size effect* is the effect in which the adoption of a given class of consumers is pivotal to other consumers adopting the product. E.g., if Dropbox would seize to offer the free-of-charge contract with 2 GB of storage, the market size effect translates into the number of subscribers of the premium contracts leaving the service as a consequence of not being able to share content with subscribers of the free contract. Second, the *distribution effect* is the effect in which the adoption of a given class of consumers incentivize other consumers to upgrade to a higher contract. In the Dropbox example, the distribution effect translates into the number of subscribers who would downgrade to a cheaper contract if Dropbox would seize to offer the free-of-charge contract. In Appendix B I analyze friendship networks among villagers in Karnataka, India, based on data from Banerjee et al. (2013), and show that these two effects may indeed be of great importance.

For network structures where scarcely connected consumers are detrimental to the participation of all other consumers, a monopolist entering the market has no choice but to sell to a large set of market segments. In that situation the monopolist may suffer losses from scarcely connected consumers that exceed profits from the more connected ones. Due to asymmetric information, not all externalities can be extracted by the monopolist, and the monopolist may decide not to enter the market in the first place, despite production being socially desirable: a complete market failure.

Finally, the model gives a rationale for why free-of-charge contracts sometimes are offered in markets with local network externalities. In addition to being directly unprofitable, a free product is costly as it increases information rents given to high-demand customers. On the other hand, due to the externalities generated, the value to other consumers increases, which may benefit the seller if the costs by giving away something for free to low-demand consumers is outweighed by higher profits from high-demand consumer segments.

This work relates to previous research on price discrimination with network effects, pricing in social networks, and games on networks.

To my knowledge Candogan et al. (2012) and Bloch and Quérou (2013) are the first to study price discrimination on social networks. Under first-degree (perfect) price discrimination both find that for linear-quadratic payoff functions the optimal quantity sold to a consumer is proportional to the Bonacich (1987) centrality of that consumer.[2] Furthermore, they find that, with constant marginal costs, the unit price charged is the same for all consumers in a network, even if perfect price discrimination is feasible. Bloch and Quérou (2013) identify two opposing effects with respect to centrality and pricing: central nodes have a higher willingness to pay, but their consumption also generate most consumption externalities to other nodes, which suggests that central nodes should be subsidized. With constant marginal costs, these two effects cancel each other out. However, if costs are convex, the former effect dominates, and more central nodes are charged higher prices. Fainmesser and Galeotti (2016) build on these two papers and consider situations where the monopolist has partial information about consumers' in-degrees and/or out-degrees. They then solve for the optimal third degree price discrimination schedule assuming the monopolist can discriminate based on the partial information it has on each consumer.

The present paper differs from this literature in two central aspects. First, I study a model in which consumers' preferences are private information so that individual discrimination is not feasible, and discrimination is therefore of second-degree with a monopolist designing a menu of incentive compatible contracts. Second, in my model consumers' benefit from adjacent consumption is a function of the *binary* choice of whether neighbors purchase or not, while Candogan et al. (2012) and Bloch and Quérou (2013) consider settings where network benefits depend positively on neighbors' consumption intensity. I.e., in the present paper a consumer cares about whether his friends subscribe to the same service, not which contract the friends subscribe to.

I assume in my paper that the monopolist only has aggregate information about its potential customers. This assumption is not necessarily a literal one, but is rather an observation about the sort of information firms actually use in designing their policies. Although today's technology provides firms with detailed information about their customers, individual pricing in the manner described in the above mentioned papers is not commonplace. In online markets, we have seen several instances where powerful firms have attempted habit-based differential pricing, but have reverted these pricing policies to non-individual based pricing schemes once these practices have been unraveled.[3] An obvious issue with individual-based pricing is that rational consumers have incentives to change their behaviors if they suspect individual-based discriminatory practices, which clearly constrains firms to designing policies that are incen-

[2] The results from Bloch and Quérou (2013) and Candogan et al. (2012) do to a large extent follow from results in Ballester et al. (2006), Corbo et al. (2007) and Bramoullé et al. (2014) who all analyze network games of strategic substitutes or complements with linear-quadratic utility.

[3] A well-known example is the attempt of Amazon.com on differential pricing on DVDs in 2000. "Bezos calls Amazon experiment 'a mistake' ": https://web.archive.org/web/20001017181828/http://www.bizjournals.com/seattle/stories/2000/09/25/daily21.html

tive compatible. Moreover, firms may be cautious about pricing individually due to issues like customer outcry and unfavorable media coverage.

Second-degree price discrimination (nonlinear pricing) with global (as opposed to local) network effects have previously been studied by Sundararajan (2004a) and Csorba (2008). These papers differ as the former assumes network value and intrinsic value being separable, whereas the latter does not. While Csorba (2008) concludes that network effects always increase consumption, the results from Sundararajan (2004a) depend on how an increase in private valuations affects the consumers' network values and intinstic values, respectively. On a more general level these papers, as well as the present one, deal with contracting with externalities which most notably is treated in Segal (1999). To the best of my knowledge, the present paper is the first to model nonlinear pricing with local network effects.

The literature on (non-discriminatory) monopoly pricing in social networks can roughly be divided into a class of models with consumption externalities (as in the present paper) and a class of models of diffusion and influence between connected consumers. Within the first strand of literature Sääskilahti (2015) studies a static model, while Shin (2015) considers a dynamic model with myopic consumers. Within the second strand of literature Galeotti and Goyal (2009), Campbell (2013), Carroni and Righi (2015), and Ajorlou et al. (2015) study optimal selling strategies with word-of-mouth effects and other forms of influence or diffusion in social networks. Duopoly models with local network effects are considered in Banerji and Dutta (2009) and Chen et al. (2015), where the latter extends the work of Candogan et al. (2012) and Bloch and Quérou (2013). Outside the realm of social networks, Fjeldstad et al. (2015) consider monopolistic pricing with consumers distributed over a Salop circle. Local network effects are understood as utility depending positively on adoption of consumers with similar preferences. They find that equilibrium outcomes are inefficient both with a single price and with location-specific price discrimination since consumers do not fully internalize the positive externalities they impose on others by adopting

This paper also relates to work on games on networks. When payoffs depend positively on actions taken by one's connections, consumers' actions are strategic complements. Adoption games are studied in among others Jackson and Yariv (2007) and Sundararajan (2007). In the present paper, a simple approach is used with consumers playing a *threshold game of complements* in a setup similar to the one presented in Jackson (2008). I show that under a maximal and Pareto dominant Nash equilibrium, there are identifiable properties of the network structure that determines the set of contracts offered by the monopolist. Threshold games of complements are special cases of games of strategic complements where the increase in payoffs by taking a higher action increases in the actions of other players. Games of strategic complements are closely related to games of strategic substitutes in which best-response actions are decreasing in the action taken by others (e.g., public good games). Several variations of such games are treated in among others Galeotti et al. (2010) and Jackson and Zenou (2014).

The rest of the paper is structured as follows: Part 2 sets up the main ingredients of the model, and part 3 provides a benchmark model where the monopolist contracts with exogenous types. The main model where types depend on actions taken in the network is presented in part 4. Part 5 concludes. Extensions and an empirical application are found in the Appendices.

2 Model preliminaries

There is a finite set of consumers, $I = \{1, 2, \ldots, n\}$. The connections between the consumers are represented by a social network described by an $n \times n$ adjacency matrix $\boldsymbol{G}$. An element g_{ij} of $\boldsymbol{G}$ describes the connection between nodes i and j. The network is assumed to be undirected and unweighted,[4] i.e., $g_{ij} \in \{0, 1\}$ for all $i, j \in I$, where $g_{ij} = g_{ji} = 1$ implies that consumers i and j are connected, while i and j are not connected whenever $g_{ij} = g_{ji} = 0$. No consumers have self links, i.e., $g_{ii} = 0$ for all $i \in I$.[5] The neighborhood of consumer i, $N_i(\boldsymbol{G})$, is the set of all agents who are connected to i: $j \in N_i(\boldsymbol{G}) \Leftrightarrow g_{ij} = 1$.

The *network degree* of agent i, $\tilde{d}_i(\boldsymbol{G})$, is the number of i's connections:[6]

$$\tilde{d}_i(\boldsymbol{G}) = \sum_{j=1}^{n} g_{ij}.$$

The set of network degrees in network $\boldsymbol{G}$ is $\{0, 1, \ldots, \tilde{m}\}$, where $\tilde{m} \leq n - 1$. Without loss of generality consumers with no connections are disregarded. Hence, the set of network degrees is limited to $\tilde{D} = \{1, 2, \ldots, \tilde{m}\}$.

Each consumer i takes an action $b_i \in \{0, 1\}$, where $b_i = 1$ implies that i buys the product.[7] The set of actions taken by all consumers is represented by a vector $\boldsymbol{b} = (b_1, b_2, \ldots, b_n)$. The *degree* of agent i, $d_i(\boldsymbol{b}; \boldsymbol{G})$, is defined as the number of buyers in the neighborhood of i:

$$d_i(\boldsymbol{b}; \boldsymbol{G}) = \sum_{j=1}^{n} g_{ij} b_j.$$

Hence, this paper departs somewhat from the standard vocabulary in the social network literature: What I call *degree* is in fact the *buyer degree* (the type of degree consumers care about), while *network degree* is what traditionally is referred to as *degree* in the social network literature.

A consumer's degree is also his privately observed and endogenously determined *type*. This type determines the function $\theta_i = \theta(d_i)$, which is the privately observed part of a consumer's utility. Local network effects are captured by the value of consumption increasing in a consumer's degree:

$$\theta(d_i + 1) > \theta(d_i).$$

[4] An extension with weighted links is considered in Appendix A.

[5] Self links, interpreted as network independent valuations, are considered as an extension in Appendix A.

[6] This is usually referred to as simply the "degree". It will be clear shortly why a distinction between "network degree" and "degree" is made.

[7] The quantity (or quality), q, of the product bought may differ between individuals. We set $b_i = 1$ if $q_i > 0$.

Hence, the privately observed valuation of a given consumer is a function of the actions taken in his neighborhood. Two consumers with the same degree value consumption equally, and valuation is therefore independent of the consumer's identity. For the remainder of the paper I write $\theta_d = \theta(d)$ for any consumer with degree d.

The probability of a randomly selected agent having degree d (equivalently, having valuation θ_d) is defined as:

$$\beta(d) = \Pr(d|\boldsymbol{b}, \boldsymbol{G}),$$

which is distributed according to a probability density function $\phi(d)$ with the associated cumulative distribution function $\Phi(d)$. Since each consumer's type is determined by the actions taken by one's neighbors, the degree distribution, $\Phi(d)$, is endogenous as it depends on the actions taken in the network. Following the above slight abuse of notation, I write $\beta_d = \beta(d)$ for the remainder of the paper.

The net utility of a consumer with degree d has the following functional form:

$$U_i(q, P, d) = \begin{cases} \theta_d v(q) - P - f & \text{if } b_i = 1 \\ 0 & \text{if } b_i = 0, \end{cases} \tag{1}$$

where q is the quantity (or quality) of the good obtained, P is the price paid for a given quantity, and $v(q)$ is a function that satisfies standard properties of diminishing marginal returns and interior solutions: $v' > 0, v'' < 0, v(0) = 0$, and $\lim_{q \to 0} \theta_1 v'(q) > c$, where c is the marginal cost of production. Because of the quasi-linear functional form, the valuation of a given quantity, $\theta_d v(q)$, is measured in currency. The parameter $f \geq 0$ is a transaction cost imposed on the consumer by obtaining the product. This can be interpreted as a cost associated with learning how to use the product, or as the value of an outside option assumed to be the same for all consumers (e.g. sharing files over e-mail instead of using a cloud service).[8] Since network benefits from obtaining the social good depend only on the binary choice taken by one's neighbors and not the quantities consumed, the monopolist could offer a virtually costless quantity $q = \epsilon \to 0$ to ensure participation among low-demand consumers. The parameter f ensures that such behavior is costly for the seller as consumers must be compensated for the transaction cost.

Since a consumer's type depends positively on the number of buyers in one's neighborhood, consumption exhibits direct positive externalities, i.e., the marginal benefit from q is increasing in neighbors' actions. For $d > d'$:

$$\frac{\partial U(q, P, d)}{\partial q} > \frac{\partial U(q, P, d')}{\partial q}.$$

Actions are also *strategic complements*, i.e., a consumer's change in payoff by increasing one's action from $b_i = 0$ to $b_i = 1$ is increasing in neighbors' actions. For $d > d'$:[9]

[8] A similar assumption is made in Sundararajan (2007). One can also interpret f as a fixed cost imposed on the seller.

[9] The general definition of strategic complements is stated in e.g. Jackson and Zenou (2014): For all i, $a_i \geq a_i'$ and $a_{-i} \geq a_{-i}'$:

$$u_i(a_i, a_{-i}, \boldsymbol{G}) - u_i(a_i', a_{-i}, \boldsymbol{G}) \geq u_i(a_i, a_{-i}', \boldsymbol{G}) - u_i(a_i', a_{-i}', \boldsymbol{G}).$$

$$\theta_d v(q) - 0 > \theta_{d'} v(q) - 0.$$

Finally, the assumptions on θ and $v(q)$ ensure that the Spence-Mirrlees single-crossing condition is satisfied:

$$\frac{\Delta}{\Delta d}[-\frac{\partial U/\partial q}{\partial U/\partial P}] = \frac{\Delta\theta}{\Delta d}v'(q) > 0.$$

3 Benchmark model: Exogenous types

As a benchmark this section presents a standard textbook model where types are exogenous, and the monopolist offers a set of incentive compatible contracts resulting in a separating equilibrium where consumers of different types self-select into a contract intended for their type. For sake of comparison in the subsequent parts, I assume in this section that the distribution of types is equivalent to the *network degree* distribution in the social network, and types are therefore exogenously given and determined by their exogenously given network degree, $\tilde{d}_i$. For instance, in this benchmark model a node with two links is type 2, with gross utility $\tilde{\theta}_2 v(q) - f$, regardless of whether his connections have obtained the good or not.[10] The model in this section is a version of the one provided in Bolton and Dewatripont (2005), where the only extension is the transaction cost $f \geq 0$ imposed on the participating consumers, as well as the type distribution being determined by the network degree distribution in an exogenously given network $\boldsymbol{G}$.

Assumption 1. *The monopolist has full information of the structure of the network* $\boldsymbol{G}$*. The identity (location) of each individual on the network is unknown to the monopolist.*

For exogenous types, the assumption is equivalent to each individual's type being private information, but the distribution of types being known to the monopolist. Hence, the monopolist cannot perfectly discriminate over the network. However, it can offer a set of contracts the consumers can select from. Degrees are distributed over a probability density function $\phi(\tilde{d})$ with the associated cumulative density function $\Phi(\tilde{d})$. The *Mills ratio*, $M(\tilde{d})$, is the reciprocal of the *hazard ratio* interpreted as the share of nodes with degrees exceeding $\tilde{d}$ relative to the share of nodes with degree $\tilde{d}$:

$$M(\tilde{d}) = \frac{1 - \Phi(\tilde{d})}{\phi(\tilde{d})} = \frac{1 - \sum_{i=1}^{d} \tilde{\beta}_i}{\tilde{\beta}_d}.$$

From the Mills ratio, the following assumption is imposed.

Assumption 2.

$$\frac{\Delta}{\Delta \tilde{d}}\left[\tilde{\theta}_d - M(\tilde{d})\Delta\tilde{\theta}_d\right] > 0$$

for all $\tilde{d}$*, where* $\Delta\tilde{\theta}_d = \tilde{\theta}_{d+1} - \tilde{\theta}_d$.

[10] Parameters that are exogenous in the benchmark model which are endogenized in the main model are denoted with a tilde. For cosmetic reasons, subscript letters are given without a tilde: $\tilde{\theta}_d = \tilde{\theta}_{\tilde{d}}$.

Assumption 2 is slightly less restrictive than assuming a decreasing Mills ratio (equivalently increasing hazard ratio). The assumption ensures that equilibrium quantities are increasing in types so that the maximization problem subject to local incentive compatibility constraints is sufficient, and is normally satisfied for most unimodular distributions.[11] This simplifying assumption ensures that "bunching" (local pooling) of types following the ironing procedure by Myerson (1981) is not implemented.

The monopolist faces the cost function $C(q) = cq$, and solves the following problem subject to local constraints:[12]

$$\max_{k,\{(P_d,q_d)\}} \sum_{d=k}^{\tilde{m}} [P_d - cq_d]\tilde{\beta}_d.$$

subject to:

$$\tilde{\theta}_k v(q_k) - P_k - f \geq 0 \tag{PC}$$

$$\tilde{\theta}_d v(q_d) - P_d \geq \tilde{\theta}_d v(q_{d-1}) - P_{d-1} \text{ for all } \tilde{d} > k \tag{IC}$$

$$q_m \geq q_{m-1} \geq \cdots \geq q_k \tag{QQ}$$

Following Assumption 2 the constraint (QQ) is automatically satisfied. The parameter k denotes the lowest type offered a contract. Hence, if $k = 1$, we have full participation. If $k \geq 2$ then nodes of degree $k - 1$ and lower are not offered a contract satisfying their participation constraints. I return to the choice of k shortly. First, for a given k the first order conditions are:

$$\theta_d v'(q_d^{SB}) = \frac{c}{1 - M(\tilde{d})\dfrac{\Delta\tilde{\theta}_d}{\tilde{\theta}_d}} \text{ for } k \leq \tilde{d} \leq \tilde{m}. \tag{2}$$

Since $M(\tilde{m}) = 0$, q_m is the only efficiently offered quantity as marginal benefit equals marginal cost. The monopolist distorts the quantity downwards for all types less than $\tilde{m}$ in order increase profits from higher types. From equation (2) it is clear that quantities are increasing in types following Assumption 2. The set of profit maximizing prices are found from (PC) and (IC) in the maximization problem:

$$P_k = \tilde{\theta}_k v(q_k^{SB}) - f \tag{3}$$

$$P_d = \tilde{\theta}_d v(q_d^{SB}) - f - \sum_{i=k}^{d-1} \Delta\tilde{\theta}_i v(q_i) \text{ for } \tilde{d} \geq k+1, \tag{4}$$

where $\sum_{i=k}^{d-1} \Delta\tilde{\theta}_i v(q_i)$ is the information rent given to types $\tilde{d} > k$. For notational ease, I will throughout the rest of the paper refer to q_d^{SB} as q_d defined by equation (2), unless otherwise stated.

[11] E.g., the increasing hazard rate property of the degree distribution is satisfied for Poisson random graphs, but not for networks generated by strong preferential attachment (e.g. citation networks).

[12] The cost function $C(q)$ could alternatively be interpreted as a variable transaction cost to consumers as in Sundararajan (2004b), which may be more natural in a setting for information goods where marginal costs are close to zero.

3.1 The lowest contract

There are two types of costs associated with participating consumers. The first are direct types of costs: production costs cq_d and transaction cost f. The second is the indirect cost of the information rent that must be offered to all other consumers with degree larger than k. For $f = 0$, the lowest participating type, k, is given by the lowest type where the inequality

$$\tilde{\theta}_k - M(k)\Delta\tilde{\theta}_k > 0$$

is satisfied. If the above inequality is violated, the first-order condition (2) yields negative quantities in equilibrium, and under some functional forms of $v(q)$, this quantity is a complex number which does not make economically sense in this setting.

Generally, with the above inequality satisfied, the profits from the lowest participating type can be written as:

$$\tilde{\beta}_k[\tilde{\theta}_k v(q_k) - cq_k - f] - [1 - \Phi(k)]\Delta\tilde{\theta}_k v(q_k).$$

The last part in the above expression is the information rent given to degrees $\tilde{d} \geq k + 1$, which otherwise would not be given if degree k nodes where not served. It is therefore profitable to serve nodes of degree k whenever the gross consumer surplus less information rents are positive, i.e, whenever the inequality

$$f \leq \tilde{\theta}_k v(q_k) - cq_k - \frac{1 - \Phi(k)}{\tilde{\beta}_k}\Delta\tilde{\theta}_k v(q_k)$$

holds. Note that $[1 - \Phi(k)]/\tilde{\beta}_k$ is the Mills ratio, $M(k)$. Inserting the expression for the profit maximizing quantity q_k from equation (2), the expression can be simplified to:

$$f \leq cq_k[\frac{v(q_k)}{q_k v'(q_k)} - 1] =: f_c(k). \tag{5}$$

Proposition 1. *When types are exogenous, the lowest participating type, k^*, is such that (i) $f_c(k^* - 1) < f \leq f_c(k^*)$, and (ii) $1 - M(k^*)\Delta\tilde{\theta}_{k^*}/\tilde{\theta}_{k^*} > 0$.*

Hence, if the lowest k which satisfies $f \leq f_c(k)$ is greater than 1, the seller excludes consumers of degrees $1 \leq \tilde{d} \leq k - 1$.

With k^* defined as in Proposition 1, the profits are given as follows:

$$\pi^* = \sum_{d=k^*}^{\tilde{m}} \tilde{\beta}_d[(\frac{1}{\alpha(q_d)} - 1)cq_d - f] \tag{6}$$

where $\alpha(q_d) = q_d v'(q_d)/v(q_d)$ is the quantity elasticity of the function $v(q)$.

4 Main model: Endogenous types

The degree, d, of each consumer depends on the number of participating neighbors, and is for that reason endogenously determined as described in Part 2. In turn, private valuations, θ_d, and the degree distribution, $\Phi(d)$, in the network are endogenously determined. Formally, the timing of events is as follows:

(a) Minimal equilibrium **(b)** Maximal equilibrium

Figure 1: Minimal and maximal Nash equilibria in a threshold game of complements with threshold 1.

1. Network $\boldsymbol{G}$ is realized.

2. Seller observes the network structure.

3. Seller announces contracts as price-quantity pairs $(P_k, q_k), (P_{k+1}, q_{k+1}), \ldots, (P_m, q_m)$.

4. Consumers simultaneously choose whether to subscribe to a price-quantity pair (P_d, q_d), $1 \leq k \leq d \leq m$ (action $b_i = 1$), or not to subscribe (action $b_i = 0$).

Since the benefit, and in effect the decision to participate, depends on the participation of those the consumers are connected to, there are multiple equilibria in step 4. As will be clear shortly, the agents are playing a *threshold game of complements*, depending on the lowest contract, (P_k, q_k), announced in step 3 above. This is a special type of coordination game in which player i chooses action $b_i = 1$ (buy) only if at least k of i's neighbors take the same action.

Definition. *In a threshold game of complements with threshold k in network $\boldsymbol{G}$, $\mathcal{T}(k;\boldsymbol{G})$, and payoff functions $U_i(b_i, d_i(\boldsymbol{b};\boldsymbol{G}))$, payoffs are such that:*

$$U_i(1, d_i(\boldsymbol{b};\boldsymbol{G})) \geq U_i(0, d_i(\boldsymbol{b};\boldsymbol{G})) \text{ if and only if } d_i(\boldsymbol{b};\boldsymbol{G}) \geq k.$$

A pure strategy Nash equilbrium in the game $\mathcal{T}(k;\boldsymbol{G})$ is a profile of strategies $\boldsymbol{b} = (b_1, b_2, \ldots, b_n)$ such that:

$$b_i = 1 \text{ if } d_i(\boldsymbol{b};\boldsymbol{G}) \geq k, \text{ and}$$

$$b_i = 0 \text{ if } d_i(\boldsymbol{b};\boldsymbol{G}) < k.$$

A threshold game of complements in networks typically have multiple equilibria. Figure 1 illustrates that even in a simple network with two connected nodes, there are two equilibria with zero and full participation, respectively. In order to limit the number of outcomes, the following assumption is imposed:

Assumption 3. *The realized Nash equilibrium in a threshold game of complements, $\mathcal{T}(k;\boldsymbol{G})$, is the maximal equilibrium, i.e., the one with the highest level of participation.*

While successful coordination is the Pareto dominant outcome, the maximal equilibrium assumption is a strong one for one-shot games. In addition to favorable expectations on neighbors' actions, the assumption implies full information about the network structure and own position in the network for $k \geq 2$. A one-shot coordination to the maximal equilibrium would also require an unusually high level of sophistication among consumers.

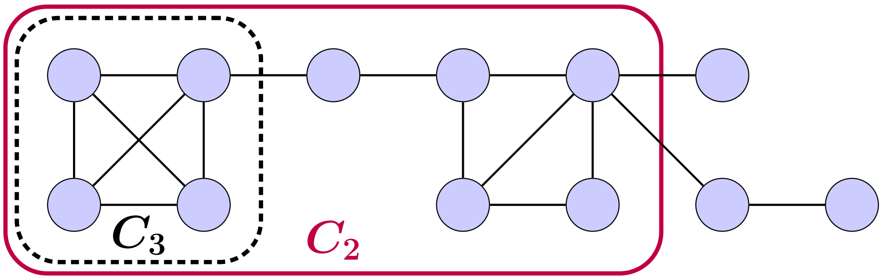


Figure 2: Nodes contained in the solid box are in C_2, nodes in the dashed box are in C_3. Nodes that are in a C_3 are also in C_2. All nodes in the network are in C_1.

The argument for realization of the maximal equilibrium is a dynamic one. In Appendix A I show that by adding dynamics and adaptive expectations on a consumer's neighbors' actions, the maximal Nash equilibrium is implemented in finite time. Thus, the optimal contracts described in this section can be interpreted as the optimal per-period contracts of a perishable good for any period following the time when the maximal Nash equilibrium is reached.

As this is a game of strategic complements, identifying the maximal equilibrium is a straightforward exercise solved by the following algorithm (see e.g. Jackson and Zenou (2014)): Let all nodes take the high action in an initial state, $b_i = 1$ for all i, and then solve for each node's best-response iteratively. In the first step, all nodes having a degree less than k, $d_i < k$, change their actions to $b_i = 0$. If at least one node reduced its action in the first step, the process is repeated in a second step where nodes with $d_i < k$ have $b_i = 0$ in the initial state. We repeat this process until we reach a state in which no further nodes decrease their actions.

The maximal Nash equilibrium for the game $\mathcal{T}(k; \boldsymbol{G})$ depends on the structure of the network $\boldsymbol{G}$. A necessary condition for participation for a threshold of $k \geq 2$ is that the network contains a *cycle*, a sequence of links connecting a sequence of nodes that starts and ends at the same node. The general requirement for non-zero participation at threshold k is the existence of what I define as a k-cycle, C_k.

Definition. *C_k is a collection of nodes who take action $b_i = 1$ in the maximal Nash equilibrium in a threshold game of complements with threshold k.*

C_k is therefore the set of nodes that survive the previously described algorithm. Figure 2 illustrates nodes in C_2 and C_3. Starting with full participation, the three nodes outside C_2 drop out at a threshold game of complements with threshold 2, while the rest of the nodes still have at least two adjacent nodes participating, and are therefore in a C_2 per definition. One of the nodes outside C_2 has a network degree of 2. However, one of its adjacent nodes are of degree 1, leaving the node with only one adjacent buyer. Thus, this node drops out at threshold 2.

Note that the degree-2 node located in the middle of the C_2 is *not* in a cycle, despite being in a C_2. However, having two separate *paths* leading to nodes in a cycle "protects" this node from sequential elimination of nodes of degree 1.[13] At threshold 3, all nodes except for the group of

[13] A *path* is a sequence of links connecting a sequence of nodes in which each node appears at most once in the sequence.

four to the left drop out. Therefore, having many links is not sufficient for participation. In C_3 three of the nodes have exactly three links, while the top right node in C_2 have 5 links, but does not participate at threshold 3 since less than three (zero) of its neighbors do. Note that the set of nodes in C_3 also are contained in C_2 and so on: $C_m \subseteq C_{m-1} \subseteq \cdots \subseteq C_1$.

Since the type of each consumer now is determined endogenously, Assumption 2 from the benchmark case is modified:

Assumption 4. *For any maximal Nash equilibrium in a threshold game of complements, $\mathcal{T}(k; G)$, the following inequality holds for all d.*

$$\frac{\Delta}{\Delta d}\Big[\theta_d - M(d)\Delta\theta_d\Big] > 0$$

As in Part 3 the above assumption ensures that local incentive compatibility is sufficient for obtaining a solution to the monopolist's maximization problem, i.e., bunching of some types is not implemented in equilibrium.

A profit maximizing monopolist facing a constant marginal cost, c, now solves the following problem:

$$\max_{k,\{(P_d,q_d)\}} \sum_{d=k}^{m} [P_d - cq_d]\beta_d$$

s.t.

$$\theta_k v(q_k) - P_k - f \geq 0 \tag{PC}$$

$$\theta_d v(q_d) - P_d \geq \theta_d v(q_{d-1}) - P_{d-1} \text{ for all } d > k \tag{IC}$$

$$q_m \geq q_{m-1} \geq \cdots \geq q_k \tag{QQ}$$

$$d_i = d_i(k, \boldsymbol{G}) = d_i(\mathcal{T}(k; \boldsymbol{G})) \Rightarrow \theta_{id} = \theta_{id}(k),\ \beta_d = \beta_d(k) \tag{KK}$$

As before, k is the node with the lowest degree that is offered a contract satisfying individual rationality. The solution to the above maximization problem at a given k is equivalent to the first order conditions in the benchmark with exogenous types, and is omitted for preservation of space. However, if full participation is not implemented, the distribution of types may differ between the two cases. With $M(d) \neq M(\tilde{d})$ for some d, the optimal contract yields a different quantity and price to a given type in the two cases.

The main difference from the benchmark with exogenous types is that the monopolist must take into account the maximal Nash equilibrium from the threshold game of complements, $\mathcal{T}(k; \boldsymbol{G})$. The condition (KK) emphasizes that the types, d, and the degree distribution, represented by shares β_d, depend on the lowest participating type, k, through the realized equilibrium of the threshold game of complements.

Negative prices are allowed for. This may be unrealistic as consumers could take the cash without using the product, unless there is a commitment mechanism to consume (i.e. not incurring the transaction cost f, which might be interpreted as switching from the outside option).

Hence, the "cash takers" would not generate any externalities to consumers in their neighborhood. The case with prices being bounded to non-negative values is considered in Appendix A.

Before proceeding to identification of the set of profit maximizing contracts, we have the following trivial, albeit not unimportant, result:

Proposition 2. *If the optimal contract for exogenous types has full participation, $k^* = 1$, then the optimal contract with endogenous types is also with full participation, $k^* = 1$.*

Proof. With the assumption on the maximal Nash equilibrium being imposed in $\mathcal{T}(1; \boldsymbol{G})$, network degree equals degree for all individuals: $\tilde{d}_i = d_i$ for all i. Hence, the payoff for all consumers are equal in the two cases, and the maximization problems for the monopolist are identical in the two cases. □

In order to properly explore the workings of price discrimination on social networks, the rest of the paper focuses on the situation where full participation is *not* optimal for exogenous types.

4.1 The lowest contract

Exclusion of consumers occurs whenever the monopolist set $k \geq 2$, i.e., consumers of degree 1 are not offered a contract.[14] If exclusion is implemented, the set of participating nodes is defined by those who take the action $b_i = 1$ in the maximal equilibrium of $\mathcal{T}(k; \boldsymbol{G})$. Participation of a given node depends on its position in the network $\boldsymbol{G}$, summarized by the following proposition:

Proposition 3. *If the lowest contract that satisfies the participation constraint is for type k, then all nodes in C_k participate, while all nodes not in C_k do not participate.*

Proof. The lowest contract offered is (P_k, q_k), where P_k for a given q_k is determined by the equality $\theta_k v(q_k) - P_k - f = 0$. Thus, for $l \in \{1, \ldots, k-1\}$ a consumer of degree $k - l < k$, i.e., with private valuation $\theta_{k-l} < \theta_k$, has his participation constraint violated at the lowest contract offered: $\theta_{k-l} v(q_k) - P_k - f < 0$. By definition, in a maximal equilibrium of $\mathcal{T}(k; \boldsymbol{G})$ all participating nodes are in C_k and have degrees k or higher. □

Network degrees, $\tilde{d}_i$, and *degrees*, d_i may differ substantially when full participation is not implemented. All nodes not in C_k have their degrees reduced to zero by exclusion. Hence, potential high-degree consumers may drop out of the market. Figure 3 illustrates an example with a *star* component where the central node has a potential degree of 5, and in turn a high benefit of participating if his neighbors, all of degree 1, participate. But with exclusion, i.e., any $k \geq 2$, then the central node drops out.

In addition, nodes who are in C_k and therefore choose to participate, have their degrees reduced if they are connected to nodes outside C_k relative to a case where degrees of $k - 1$ and

[14] Nodes of network degree 1 is assumed to exist in the network $\boldsymbol{G}$. Generally, exclusion of consumers is implemented whenever $k > \min \tilde{D}$.

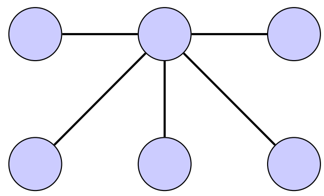


Figure 3: A star: The central node has a potential degree of 5, but will only participate under full participation.

higher participate. In short, the degree of a node i, and in return its valuation, θ_i, may be reduced if exclusion is implemented:

$$\frac{\Delta d_i}{\Delta k} \leq 0.$$

The following subsection addresses the effects by exclusion in detail.

4.1.1 Properties of location and network structure

When consumers of degrees $k-1$ and lower are excluded, the magnitude on the reduction in other consumers' degrees ultimately depends on the network structure of $\boldsymbol{G}$. In order to analyze the effects further, I classify three key attributes of the location of a node i. Three key subsets of $i \in G$, denoted X_k, Y_k, and Z_k, are defined as follows:

1. Nodes that are in C_k and are only adjacent to nodes in C_k:

$$X_k = \{i \in \boldsymbol{G} : i \in C_k, (\forall j \in N_i, j \in C_k)\}.$$

2. Nodes that are in C_k and are adjacent to at least one node not in C_k:

$$Y_k = \{i \in \boldsymbol{G} : i \in C_k, (\exists j \in N_i, j \notin C_k)\}.$$

   This set can be divided into further subsets $Y_{k\#}$ where $\# \in \{1, 2, \cdots, l\}$ is the number of i's neighbors who are not in C_k.

3. Nodes that are not in C_k:

$$Z_k = \{i \in \boldsymbol{G} : i \notin C_k\}.$$

For $i \in X_k$, excluding nodes of degree $k-1$ affects neither participation nor the gross benefit for a given quantity, and so the degree distribution within this set of consumers remains the same after exclusion. For $i \in Y_k$, exclusion reduces the degree of each i by at least one unit, so that the willingness to pay is reduced in this segment, but not to the extent that these nodes drop out. This is the distribution effect by exclusion, as the degree distribution within this segment is changed. For $i \in Z_k$ the degree is less than k for all nodes following exclusion, thus none in this segment participates. This is the market size effect by exclusion.

Define, for any variable ω_d associated with degree d, its associated value $\omega_{d|k}$ when k is the lowest participating degree. The share of the nodes who have degree d and are in X_k, Y_k or Z_k

when k is the lowest contract are denoted $\beta_{d|k}^{X_k}, \beta_{d|k}^{Y_k}, \beta_{d|k}^{Z_k}$, respectively. For simplicity I assume the utility elasticity w.r.t. q to be constant, i.e., the function $v(q)$ in (1) has the form $v(q) = \sigma q^\alpha$ where $0 < \alpha < 1$ and σ a positive constant. The change in profits by increasing the lowest contract from k to $k+1$ is then:

$$\begin{aligned}
\pi_{k+1} - \pi_k = (\frac{1}{\alpha} - 1)c\Bigg[& -\beta_{k|k}[q_{k|k} - \hat{f}] \\
& + \sum_{d=k+1}^{m} \beta_d^{X_{k+1}}[q_{d|k+1} - q_{d|k}] \\
& + \underbrace{[\sum_{d=k+1}^{m} \beta_{d|k+1}^{Y_{k+1}} q_{d|k+1} - \sum_{d=k+1}^{m} \beta_{d|k}^{Y_{k+1}} q_{d|k}]}_{\text{Distribution effect } (<0)} \\
& \underbrace{-[\sum_{d=k+1}^{m} \beta_{d|k}^{Z_{k+1}} q_{d|k} - \hat{f}]\Bigg]}_{\text{Market size effect } (<0)},
\end{aligned} \tag{7}$$

where $\hat{f} = f/[(1/\alpha) - 1]c$. If profits increase by increasing the lowest contract, then equation (7) is positive. For a maximum of $\pi(k)$ when evaluated at the profit maximizing pairs of $(q_d(k), P_d(k))$, the profit maximizing level of participation, k^* is given by the solution of the following problem:

$$k^* = \arg\max_k k \text{ s.t. } \pi_{k+1} - \pi_k \leq 0. \tag{8}$$

That is, k^* is the highest k in which the first difference in (7) is non-positive, meaning it is not optimal to increase the lowest contract further. At k^* we can solve for the direct profits obtained from the lowest contract:

$$\begin{aligned}
\beta_{k^*}[(\frac{1}{E} - 1)cq_{k^*} - f] = & \underbrace{\pi_{k^*} - \pi_{k^*+1}}_{\geq 0} \\
& + \underbrace{(\frac{1}{\alpha} - 1)c\Bigg[\sum_{d=k^*+1}^{m} \beta_d^{X_{k^*+1}}[q_{d|k^*+1} - q_{d|k^*}]}_{\lesseqgtr 0} \\
& + \underbrace{[\sum_{d=k^*+1}^{m} \beta_{d|k^*+1}^{Y_{k^*+1}} q_{d|k^*+1} - \sum_{d=k^*+1}^{m} \beta_{d|k^*}^{Y_{k^*+1}} q_{d|k^*}]}_{\text{Distribution effect } (<0)} \\
& \underbrace{-[\sum_{d=k^*+1}^{m} \beta_{d|k^*}^{Z_{k^*+1}} q_{d|k^*} - \hat{f}]\Bigg]}_{\text{Market size effect } (<0)}.
\end{aligned} \tag{9}$$

Direct profits from lowest type is a non-negative term, $\pi_{k^*} - \pi_{k^*+1}$, in addition to three terms that are interpreted as the local network externalities accrued to the monopolist from participation of consumers of degree k^*. If the lowest contract is increased to k^*+1, these three terms represent changes in profits from nodes located in X_{k+1}, Y_{k+1}, and Z_{k+1}, respectively.

The change in profits from the consumer segment in X_{k+1} is ambiguous, and the change is due to changes in the degree distribution, which affects the optimal quantity offered to each d by equation (2).[15] From the consumer segment in Y_{k+1}, where all nodes have their degree decreased, it is clear that among nodes with degree k or higher, the degree distribution with k stochastically dominates the distribution without k, i.e.,

$$\sum_{i=k+1}^{d} \beta_{d|k+1}^{Y_{k+1}} > \sum_{i=k+1}^{d} \beta_{d|k}^{Y_{k+1}} \text{ for all } d < m.$$

Hence, the consumers in this segment all have a lower benefit from consumption when removing the contract to degree-k nodes, yielding lower profits to the monopolist. In the final segment of consumers outside C_k all profits are lost, despite this segment having a large potential profits.

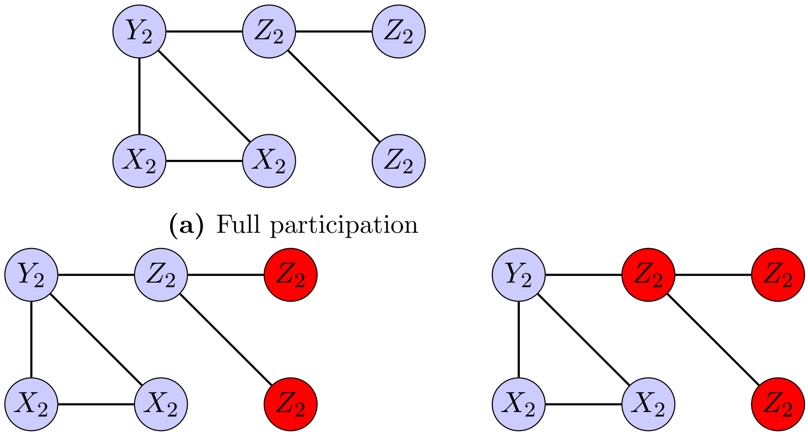
Thus, it is indeed possible that the monopolist derives direct negative profits from the lowest participating type, k^*. If this is the case, the profits lost from other consumers by increasing the lowest contract to $k^* + 1$ must be greater than the foregone losses of excluding consumers of degree k^*.

Example

A simple example illustrating the effect of exclusion of degree 1 nodes in a network, i.e., going from $k = 1$ to $k = 2$, is illustrated in Figure 4. three of the nodes are in Z_2: at $k = 1$ two of these nodes have degree 1, and one node has degree 3. Of the remaining nodes who are in a C_2 (equivalent to a cycle in this example), the degree-3 node is in Y_2 since it is connected to one node outside the cycle. The two remaining nodes are of degree 2 and are only connected to nodes in the cycle and therefore belong to subset X_2.

With full participation, the degree distribution is $\boldsymbol{\beta} = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$, with $\sum \boldsymbol{\beta} = 1$. Suppose $(\frac{1}{\alpha} - 1)cq_1^{SB} - f < 0$, i.e. serving nodes of $d = 1$ yields negative direct profits when evaluated at the profit maximizing quantities given $k = 1$. When types are exogenous, exclusion only leads to unprofitable nodes of network degree 1 dropping out, and profits are increased by $\frac{1}{3}|(\frac{1}{\alpha} - 1)cq_1^{SB} - f|$. However, when types depend on participation in a node's neighborhood, we have that, in addition to the two degree-1 nodes, the degree-3 node in Z_2 drops out, and the degree-3 node in Y_2 has its degree reduced by one unit, making this node less profitable to the monopolist. Therefore, when excluding nodes with $d = 1$ the remaining degree distribution is $\boldsymbol{\beta} = (0, \frac{1}{2}, 0)$. It is therefore not clear whether excluding degree-1 consumers increases profits, despite having negative direct profits from this segment.

[15] Numerical results with Poisson random graphs (Part 4.3) suggests that this effect is positive since increasing the lowest contract shifts the the Mills ratio down. Intuitively, exclusion typically leads to relatively higher shares of low-degree nodes due to the distribution effect. In turn the monopolist optimally puts higher weight on these consumers. However this result is not general since the market size effect may disproportionately affect low-degree nodes which may counteract the distribution effect on the full degree distribution.



(a) Full participation

(b) Exclusion with exogenous types

(c) Exclusion with endogenous types

Figure 4: Exclusion of consumers with degree 1.

4.2 Complete market failure

Since participation of a given node yields positive externalities to those in its neighborhood, the monopolist may choose to serve unprofitable nodes in order to profit from increased valuations in the neighborhood of those nodes. Moreover, following Proposition 3, the monopolist is unable to exclude consumers of degree k if no nodes are in C_{k+1}, since exclusion would lead to zero participation. Therefore, the monopolist may face an "all-or-nothing" trade-off between serving a large share of unprofitable customers and not to produce at all.

This mechanism leads to the possibility of a complete market failure. Despite a given production level being socially desirable, the monopolist chooses not to produce at all if there are negative profits at all levels of participation. I.e., the seller may incur losses from some consumer segments and profit from other consumer segments, but is not able to exclude the unprofitable segment since exclusion reduces profits from the potential profitable segment through the market size effect and distribution effect.

Complete market failure occurs when a given set of contracts would yield a positive social surplus but negative profits. This may occur since the consumer's surplus, in this case the information rents, is not accrued to the monopolist. The social surplus is given by total benefits to consumers less the production costs, cq, and the transaction cost per consumer, f:

$$W = \sum_{d=k}^{m} \beta_d(\theta_d v(q_d) - cq_d - f)$$

Some of the social surplus is distributed to the consumers through information rents, $r_d(q_d) = \sum_{i=k}^{d-1}(\theta_{d+1} - \theta_d)v(q_i)$, for all degrees larger than k. Thus, the monopoly profits are given by:

$$\pi = \sum_{d=k}^{m} \beta_d[\theta_d v(q_d) - cq_d - f - r_d(q_d)].$$

The condition for a complete market failure is summarized in the following proposition.

Proposition 4. *Suppose there exists a $k = \bar{k}$ such that there are nodes in $C_{\bar{k}}$, while there are no nodes in $C_{\bar{k}+1}$ in the network $\boldsymbol{G}$. Then there will be no production despite it being socially desirable if (i) the following inequality holds for each $k \leq \bar{k}$:*

$$\frac{\sum_{d=k}^{m} \beta_d[\theta_d v(q_d) - cq_d - r_d(q_d)]}{\sum_{d=k}^{m} \beta_d} \leq f, \tag{10}$$

and (ii) the following inequalities hold for at least one $k \leq \bar{k}$:

$$\frac{\sum_{d=k}^{m} \beta_d[\theta_d v(q_d) - cq_d - r_d(q_d)]}{\sum_{d=k}^{m} \beta_d} \leq f < \frac{\sum_{d=k}^{m} \beta_d[\theta_d v(q_d) - cq_d]}{\sum_{d=k}^{m} \beta_d}. \tag{11}$$

Proof. The proof follows from simple manipulations of the expressions for the social surplus and the monopolist's profits. The inequality (10) implies $\pi \leq 0$, while the inequalities (11) imply $\pi \leq 0 < W$. □

Within a certain interval of the transaction cost, f, total information rents exceed the potential monopoly losses from a positive production level. The intuition behind the result is as follows: Since no C_{k+1} is contained in the network: $C_{k+1} \notin \boldsymbol{G}$, exclusion of nodes with degree k would lead to exclusion of *all* nodes. If the losses of the unprofitable nodes exceed the profits from the rest of the nodes, a profit-maximizing monopolist would therefore choose not to produce at all.

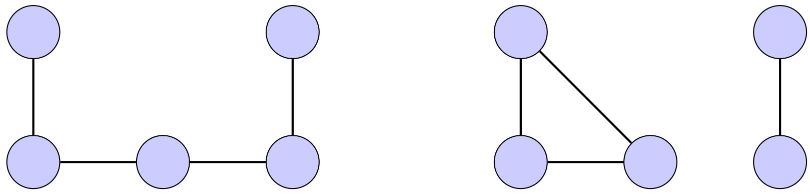
As first illustrated by Akerlof (1970), information asymmetries can lead to complete market failure when types are not separable. With separation and market power, a positive production level can be restored, but as illustrated here, the presence of network externalities can shut down the market completely, despite separability of types and no coordination failure in equilibrium.

Example

The above result is easiest illustrated in a so-called path network, the simplest network without cycles (therefore no C_2). In this type of network, full participation or no participation are the only possibilities for the monopolist as only serving nodes of degree 2 ultimately leads to zero participation, which can be easily shown through backward induction with initial drop-out of the peripheral nodes.

Figure 5 (a) illustrates a path network of 5 nodes. Under full participation there are two nodes of degree 1, and three nodes of degree 2. Thus, the distribution of degrees under full participation is $(\beta_1, \beta_2) = (2/5, 3/5)$. Suppose further that the the payoff function takes the form $v(q) = \sqrt{q}$, and put parameters at the following values: $c = 0.5, \theta_1 = 2, \theta_2 = 3$. It can be shown that under full participation the profit maximizing price-quantity pairs are $(P_1, q_1) = (1-f, 0.25)$ and $(P_2, q_2) = (8.5 - f, 9)$. From equation (6) we find that profits from each consumer segment is $\frac{2}{5}(0.125 - f)$ and $\frac{3}{5}(4.5 - f)$, which together yield total profits of $2.75 - f$.

Hence, when taking the information rent effect into account, the profits from the degree-1 segment is negative for $f > 0.125$. However, since this network is without cycles, the monopolist



(a) (b)

Figure 5: Two networks with equal degree distribution but different structure. A complete market failure can occur in network (a), but not (b).

serves all nodes as long as total profits is positive, i.e., for all $f < 2.75$. However, with $q_1 = 0.25$ and $q_2 = 9$ the social surplus is $3.05 - f$. Hence, for

$$2.75 \leq f < 3.05$$

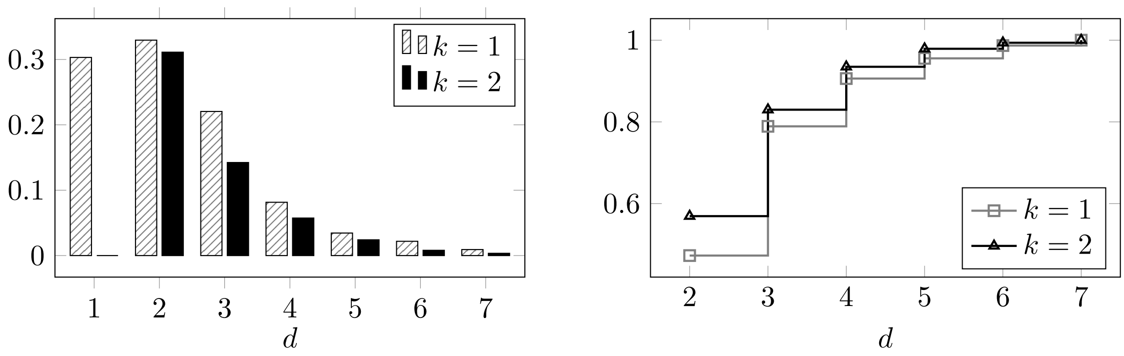
we experience a complete market failure where monopoly production would yield a positive (although not maximized) social surplus, but with negative monopoly profits. Under perfect price discrimination (which coincides with the welfare maximum), production is optimal for $f < 3.5$, and profits from degree 1 nodes are positive for $f < 2$.

As a comparison, the network in Figure 5 (b) represents a network with the exact same degree distribution, which means that networks (a) and (b) yield equal profits under full participation. However, in network (b) all the degree-2 nodes are in a cycle, and are not connected to degree-1 nodes. Thus, no externalities, direct or indirect, go from degree-1 to degree-2 consumers, and vice versa. A profit maximizing monopolist would choose to only serve nodes of degree 2 if $f > 0.125$, and total shutdown occurs at $f > 4.5$ which coincides with the critical value of f for a positive social surplus with exclusion. Thus, a complete market failure cannot occur in network (b). Note that under the assumption of a maximal Nash equilibrium, offering a menu of contracts in the network in Figure 5 (b) yields the exact same outcome as under exogenous types.

4.3 Numerical analysis: Poisson random graphs

A social network can take a vast range of topologies. Due to their simplicity and convenient properties, Poisson random graphs are widely analyzed in the social network literature and serve as a nice benchmark. In the random networks analyzed here, a network contains n nodes in which two nodes connect randomly with probability p. The degree distribution is therefore binomial, in which degrees are approximately Poisson distributed with expectation pn for large n.

In the present model, Poisson random graphs are convenient due to their statistical properties. First, Mills ratios are strictly decreasing in the Poisson distribution, which ensures that local incentive constraints are sufficient for designing a set of optimal contracts. Second, these types of networks experience sharp phase transitions for the existence of cycles at $pn > 1$



(a) Distribution of degrees

(b) $k = 1$ stochastic dominance.

Figure 6: Panel (a): Degree distribution with lowest participating degree of 1 and 2, respectively from random Poisson network with $n = 1000$ and $p = 2/n$ (degree 0 nodes omitted). Panel (b): Cumulative degree distribution conditional on $d \geq 2$. Distribution with no exclusion stochastically dominates distribution with exclusion.

(Erdős and Rényi, 1960), which also implies the existence of nodes in C_2. I provide evidence from simulations that similar phase transitions also exist for the emergence of C_k for $k \geq 3$.

In this section, I first investigate the effects on participation and distribution of degrees at different level of participation. Second, I briefly look at the implications for the monopolist's profits under specific functional forms and parameter values.

4.3.1 Statistical properties

When increasing the lowest participating degree from $k = 1$ to $k = 2$, there are two effects: First, nodes outside of C_2 choose not to buy under a threshold game of complements, i.e., the market becomes smaller. Second, some participating nodes will be of lower degrees since they are connected to nodes outside of C_2. Degree distributions of a random graph with $pn = 2$ for $k = 1$ and $k = 2$ are illustrated in Figure 6.[16] At $k = 1$ we have full participation, thus the distribution sums to 1. Panel (a) illustrates the effects on the population size. By construction, no nodes of degree 1 participate at $k = 2$, but for all $d \geq 2$ there are fewer individuals of each degree compared to $k = 1$. Thus, there are potentially fewer profitable customers left in the market. Panel (b) emphasizes the effect exclusion has on the distribution degrees. Conditional on $d \geq 2$ the distribution with $k = 1$ stochastically dominates the distribution with exclusion. Hence, there are not only fewer profitable customers, but a given consumer of $d \geq 2$ is expected to be less profitable.[17]

Another aspect of interest is the share of nodes located in C_k for different levels of k. Erdős and Rényi (1960) prove that, for $n \to \infty$, there exists a cycle (therefore also C_2) with certainty for $pn > 1$. Figure 7 shows the share of nodes located in C_k for $k \in \{2, 3, 4, 5\}$ for simulations

[16] When full participation is not implemented, degrees are no longer asymptotically Poisson distributed.

[17] Within the nodes that are in Y_2, the distribution without exclusion always stochastically dominates the distribution with exclusion. It is possible to construct networks in which the stochastic dominance property is not true for the network as a whole. Hence, stochastic dominance is not a general result.

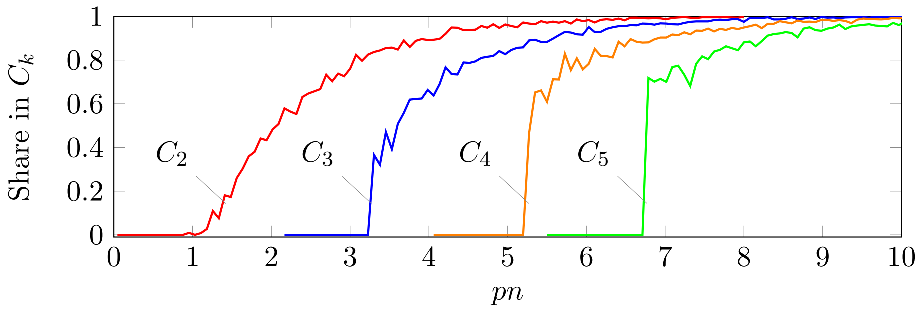


Figure 7: Share of nodes being in $C_k, k \in \{2, 3, 4, 5\}$, for given pn in Poisson random graph, $n = 1000$ in simulations.

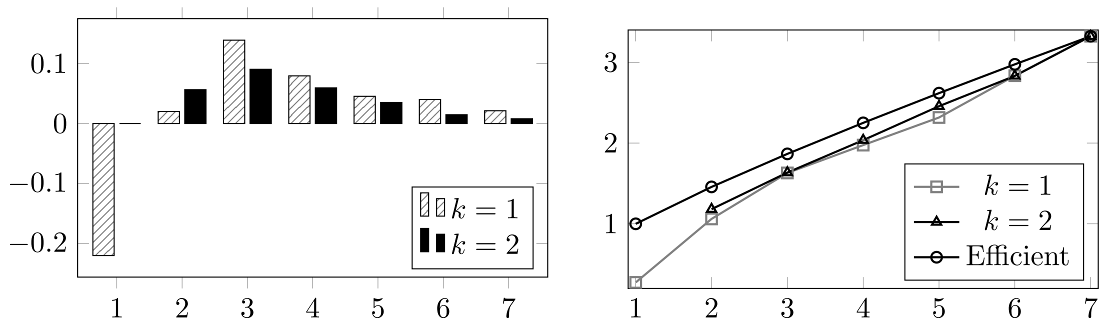
with $n = 1000$, and $pn \in [0, 10]$. The theorem of Erdős and Rényi is evident as nodes in C_2 appear approximately once pn exceeds 1. Interestingly, similar phase transitions seem to exists for $k \geq 3$. E.g. at a threshold around $pn = 3.3$ nodes in C_3 appear, and the phase transitions for higher k's appear to be more extreme with respect to the share located in C_k once the critical threshold is reached. In the previous example with $pn = 2$, around 55% of the nodes are in C_2. Despite around 43% of the participating nodes being of degrees $d \geq 3$ for $k = 2$, none are in C_3. Thus for a random graph with $pn = 2$ and a sufficiently large n, only two regimes, $k \in \{1, 2\}$, are feasible for a positive level of participation.

4.3.2 Profits and quantities

How does excluding low degree consumers affect the monopolist's profits? On one side, potential losses from low degree consumers are avoided. On the other side, the monopolist cannot benefit from the externalities they generate. For the same network depicted in Figure 6 (a), Figure 8 (a) illustrates the profits generated from each contract offered for $k \in \{1, 2\}$ for a given functional form and parameter values described in the figure caption.

When increasing the lowest participating contract, the monopolist avoids the direct negative profits from serving degree 1 consumers. On the other hand, less profits are generated from the rest of the population. This loss is interpreted as the share of externalities accrued to the monopolist from participation of degree 1 consumers.

However, exclusion also affects the distribution of degrees, which has an impact on the design of the offered contracts. In this specific numerical example, the Mills ratio shifts down following an increase in the lowest contract, leading to an increase in quantity offered to each participating type. Intuitively, there are relatively fewer consumers of high degrees, and low-degree consumers are therefore given higher weight. Thus, the monopolist finds it optimal to increase the information rents given to consumers of higher degrees, i.e., by offering higher quantities to low-demand customers, as illustrated in Figure 8 (b). In this example, profits derived from degree-2 consumers increase after increasing the lowest contract, despite there being fewer degree-2 consumers in the population. The change in the monopolist's optimal behavior therefore partially offsets the negative effects by exclusion on participation and distribution of



(a) Profits per type

(b) Quantity offered to each type

Figure 8: Parameters: $v(q) = \sqrt{q}, \theta(d) = 1 + \sqrt{d}, c = 1, f = 1$. (a) Profits, measured as gross consumer surplus less costs and information rent, per contract offered. (b) Quantity increasing in type. The first-best efficient quantities are included for comparison.

types.

5 Conclusion

How low should the monopolist go? This paper analyzed the set of profit maximizing contracts when selling vertically differentiated products exhibiting local network effects. These local network externalities are likely to cause the lowest participating type to be lower compared to a benchmark with demand being independent of one's neighbors' actions. The extent scarcely connected (low demand) consumers generate externalities to the rest of the network depends on the importance of indirect connections, which follows from identifiable properties of the network structure. Thus, when designing the menu of available contracts, the monopolist does not only care about the distribution of degrees, but also about how this distribution is affected by removing the lowest contract.

In order to analyze how a network's degree distribution is affected by removing the lowest contract, key properties of the network structure that determines the strength of local network externalities are identified. Of special importance are the share of nodes participating in a threshold game of complements (the market size effect), and within this group of nodes the share having their degree reduced by increasing the game's threshold (the distribution effect).

Furthermore, the analysis revealed that the seller has strong incentives to maintain a high level of participation, since removing the lowest contract can lead a substantial share of otherwise profitable consumers to leave the market. In relation to this, there exists a condition for a complete market failure to occur: a situation where monopoly production yields a positive social surplus, but negative private profits. Even though network externalities incentivize the monopolist to sell to a wider consumer segment, the same effects may lead to the opposite extreme in which a potential seller will not enter the market in the first place.

A Extensions

A.1 Dynamics of consumer coordination

One can easily argue that the maximal equilibrium of a threshold game of complements is unlikely to be realized in a one-shot game. In this section, I argue that with repeated purchases the maximal Nash equilibrium is attainable as a consequence of the strategic complementarities in the actions to adopt. Specifically, I consider a situation where consumers have adaptive expectations towards neighbors' adoption decisions. Although adaptive expectations are not necessarily consistent with rational expectations, the strategic complementarities ensure that adaptive expectations are consistent with fulfilled expectations once a stable equilibrium is attained. Adaptive expectations can also be used in an algorithm to identify Nash equilibria in threshold game as complements, as illustrated in Proposition A1.

With dynamics and adaptive expectations with respect to a consumers' future degree, I show that the maximal equilibrium of any k can be reached under a simple mechanism. Although implementation of the maximal equilibrium is attainable, the process of attaining the maximal equilibrium can be costly to the monopolist.

Denote the shortest path between two nodes i and j in the same component of $\boldsymbol{G}$ as its *geodesic* $\Gamma_{ij}(\boldsymbol{G})$. The following result follows:

Lemma A1. *In a dynamic threshold game of complements of threshold $k = 1$ in discrete time and adaptive expectations, $E_i[d_{it+1}] = d_{it}$, the maximal Nash equilibrium is reached at latest at time $\bar{t} = \max_{ij} \Gamma_{ij}(\boldsymbol{G})$ if*
(i) There is at least one initial adopter in each separate component of $\boldsymbol{G}$, and
(ii) The initial adopter(s) buy(s) in the first two initial periods: $t = 0, 1$.

Proof. Suppose the network $\boldsymbol{G}$ is connected. The proof can be generalized to a network with separate components as each component without loss of generality can be described as an independent network. Denote the action taken by individual i at time t as b_{it}, and fix an arbitrary node i with action $b_{i0} = b_{i1} = 1$. At $t = 1$, $b_{j1} = 1$ for all $j \in N_i$, and at $t = 2$, $b_{l2} = 1$ for all $l \in N_j$ for all $j \in N_i$. In $t = 2$, i voluntarily takes action $b_{2i} = 1$ as this is best response to actions b_{j1} for $j \in N_i$. Iterating, for later periods no players will reduce their actions, and a node i will take action $b_i = 1$ at time t if there is at least one neighbor taking the same action in $t - 1$. Thus, the time t in which a node initially adopts corresponds to the shortest path length from the initial adopter i. Full adoption is then implemented at the time corresponding to the largest geodesic from i, $t = \max_j \Gamma_{ij}(\boldsymbol{G})$, and the maximum time feasible to implement full adoption is $\max_{ij} \Gamma_{ij}(\boldsymbol{G})$. □

Since the maximal Nash equilibrium eventually can be reached at threshold 1, the lemma implies that the maximal Nash equilibrium is attainable for any k:

Proposition A1. *In a dynamic threshold game of complements of any threshold k in discrete time and adaptive expectations, $E_i[d_{it+1}] = d_{it}$, the maximal Nash equilibrium is reached in* *finite time if the monopolist increases k from an initial threshold $k = 1$ with full participation.*

Proof. Given the initial full participation, the game coincides with the algorithm to identify the maximal Nash equilibrium in games of strategic complements. See e.g. Jackson and Zenou (2014). $\square$

Lemma A1 and Proposition A1 imply that, in a dynamic setting, maximal coordination among consumers is implementable, even when we start out with zero participation. The key is that the monopolist initially offers a set of contracts in which the lowest contract satisfies individual rationality for consumers with at least one buyer in their neighborhoods. In each separate component in the network at least one consumer is "seeded",[18] which leads to full participation in finite time (Lemma A1). When full participation is implemented at $k = 1$, the desired lowest contract $k^* \geq 1$ can be implemented at the maximal equilibrium by increasing the lowest contract (Proposition A1).

A potential problem is that some consumers will have their intratemporal participation constraint (and incentive contraint) violated as some of their peers unadopt and thus are stuck with a "too high" contract in some period t_c before being able to downgrade or leave the market in $t_c + 1$, i.e., $\theta_{d-l} v(q_d) - P_d < 0$ where l is the number of unadopting friends in the preceding period. Foreseeing that the monopolist may increase the lowest contract, consumers must have their expected present value net utility satisfied before adopting a contract. The monopolist may have to commit to maintain the lowest contract at $k = 1$ for a certain amount of time, and must with certainty give the degree-1 consumers some surplus during the time before the maximal equilibrium is reached to insure them against the risk of having a one-period negative surplus.

Assume a discount factor $\delta \in [0, 1]$,[19] and denote by s_i the time i initially adopted, and $\tau_i - 1$ the time in which the last change in adoption in i's neighborhood occurred. In order to ensure full participation when the consumers expect an increase in k from 1 to $k^* \geq 2$ the following participation constraint must hold for all i:

$$E_i[\sum_{t=s_i}^{\tau_i} \delta^{\tau_i - s_i}(\theta_{idt} v(q_{dt}) - P_{dt} - f)] \geq 0,$$

$$d_{it} \geq d_t \text{ for } k = 1, \text{ and}$$

$$d_{it} \leq d_t \text{ for } k \geq 2.$$

As a consequence of adaptive expectations, the participation constraint implies possibly suboptimal intratemporal contract choices for both the consumers and the monopolist. In the adoption phase, a consumer may adopt a too low contract due to the lag following adaptive expectations. In the unadopting phase, the opposite may occur, i.e., that a consumer has a too high contract for a limited amount of time.

[18] Since the monopolist by assumption cannot identify the individuals in the network, we must assume that the monopolist publicly offers one first-come-first-serve contract of quantity q_1 at a negative price $P \leq -2f$ which lasts for two periods.

[19] With $\delta = 0$ consumers are perfectly myopic, and participation constraint satisfied in period $t = s_i$ is sufficient.

I conclude this section by noting that under less favorable assumptions on coordination in the one-shot threshold game, the maximal equilibrium is attainable by adding dynamics and adaptive expectations. The long run contracts under the maximal equilibrium with k as the lowest participating degree is equivalent to the contracts studied in Part 4. As the maximal equilibrium is not reached immediately, profits are lower compared to contracts with maximal equilibrium reached initially, due to discounting and lower participation in the interim stages of adoption. Thus, coordination failure in one-shot games is more costly to the monopolist, consumers and welfare relative to immediate successful coordination.

A.2 Non-negative prices

The assumption of allowing for negative prices may be unrealistic since consumers could take the money without adopting the product (and thereby escaping the transaction cost f). Subsequently they would not pass on the externality to potential consumers in their neighborhoods. By imposing a minimum price $P \geq 0$, consumers no longer have this incentive. Moreover, non-negative prices fit better with the empirical facts of markets analyzed in this paper. The model has shown that offering a product at a loss (including giving away for free) can be an optimal strategy due to the positive externality the low-demand consumers generate to others. Parts of the externality can be extracted by the monopolist through higher price-quantity pairs to other consumers.

A potential contract offering a quantity q_k^0 for free is such that the consumer's surplus of the lowest participating type, k, is zero, i.e.

$$\theta_k v(q_k^0) = f. \tag{A1}$$

The condition $P_d \geq 0$ is binding whenever q_d^{SB} from equation (2) implies a negative price. The quantity offered to nodes of degree d is therefore

$$q_d = \max\{q_k^0, q_d^{SB}\}.$$

Note that $P_d = 0$ may imply bunching at the bottom where consumers of degrees larger than k are offered the contract $(P_k, q_k) = (0, q_k^0)$. Monopoly profits when $P_d = 0$ and binding are lower than when negative prices are allowed, due to the increased information rent that must be given to nodes of higher degrees.

Lemma A2. *At a given k, profits are lower if the non-negativity constraint on prices binds.*

Proof. Given k and the maximal equilibrium in $\mathcal{T}(k, \boldsymbol{G})$, the value function, $\pi^*(\boldsymbol{q}^*, \boldsymbol{P}^*; k)$, is the solution to the maximized Lagrangian,

$$\mathcal{L}(\boldsymbol{q}, \boldsymbol{P}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = \sum_{d=k}^{m} \Big([P_d - cq_d]\beta_d + \lambda_{d \geq k+1}[\theta_d v(q_d) - \theta_d v(q_{d-1}) - P_d + P_{d-1}] - \mu_d [P_d - l] \Big)$$
$$+ \lambda_k [\theta_k v(q_k) - P_k - f],$$

where prices are constrained s.t. $P_d \geq l$ for all $d \geq k$ ($l = 0$ by assumption). From the Kuhn-Tucker complementary slackness condition we have that $\mu_d > 0$ if $P_d = l$, and $\mu_d = 0$ if $P_d > l$. From the envelope theorem we find the effect on profits by a marginal increase in the price floor:

$$\frac{\partial \pi^*}{\partial l} = -\sum_{d=k}^{m} \mu_d \leq 0.$$

If $P_k = l$, then $\mu_k > 0$, and possibly $\mu_d > 0$ for some $d > k$, and it follows that $\partial\pi^*/\partial l < 0$ if the price constraint binds. If the price constraint does not bind, then $\mu_d = 0$ for all d, and it follows that there is no effect on profits: $\partial\pi^*/\partial l = 0$. □

The intuition behind the above result is clear: When prices cannot be negative, and the price constraint binds, higher quantities must be given to low types in order to satisfy individual rationality. This translates into higher information rents, which reduces monopoly profits. The result also suggests that if not serving low-degree types is feasible, they are less likely to be served since a larger share of the externalities is transferred to consumers through information rents instead of monopoly profits. However, if exclusion of consumers of degree k is not feasible without excluding all consumers, lower monopolist profits increase the set of parameters in which a complete market failure occurs:

Proposition A2. *Suppose there exists a $k = \bar{k}$ such that there are nodes in $C_{\bar{k}}$, while there are no nodes in $C_{\bar{k}+1}$ in the network $\boldsymbol{G}$. If the non-negativity constraint on prices binds, then the set of values of f for which complete market failure occurs is increased.*

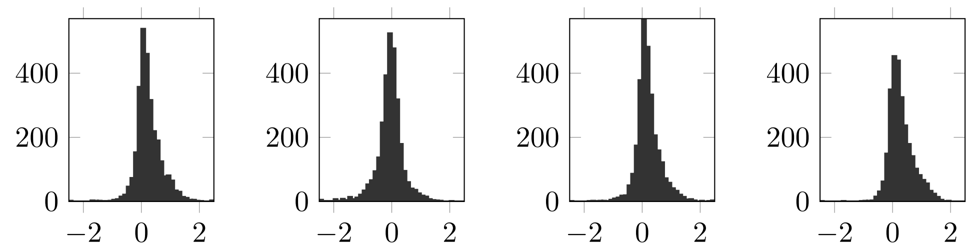
Proof. The interval of f in which a complete market failure occurs is given by inequality (11). First, note that for q_d where $P_d > 0$, quantities are not affected due to quasi-linear net utility. From Lemma A2 we know that the left-hand-side of the inequality is decreased whenever at least one price binds, i.e., the loss from increased information rents is larger than the increased profits from larger quantity-price pairs. The right-hand-side of the inequality is increased provided that $\theta_d v'(q_k^0) \geq c$, i.e. quantities for at least one type is increased towards the socially desirable quantity. □

A.3 Changes in the network structure

Together with the parameters of the model, the network structure determines the extent to which consumers of different degrees generate externalities to each other. This section aims to identify how certain alterations of a given network affects the lowest optimal level of participation, k^*. As variations in both possible networks and alterations are vast, general analytical results are difficult to obtain. The approach I follow is as follows: I conduct several simulation experiments with randomly generated networks of size $n = 40$ with a link forming at random between two individuals with probability $p = 2/n = 0.05$.[20,21] These are the "unmodified" networks, and

[20]Since nodes without links are omitted, the expected size of then network is $E(n) = 40(1-Binomial(0, 40, p = 0.05)) \approx 34.86$.

[21]The small size of the random networks was purposely chosen to ensure some variation in initial network structures for each experiment.



(a) Weighted links. Mean 0.25, $t = 29.9$

(b) Self-links (pos. and neg.). Mean -0.06, $t = -7.2$

(c) Self-links (positive only). Mean 0.21, $t = 27.1$

(d) Increased clustering. Mean 0.29, $t = 38$

Figure A1: Realizations of simulated differences in log-profits of modified networks, $\pi^M_{k=2} - \pi^M_{k=1}$, $M \in \{\textit{Weight, Self, Cluster}\}$, with parameters such that profits in unmodified networks were equal at $k = 1$ and $k = 2$: $\pi^U_{k=2} - \pi^U_{k=1} = 0$. Mean difference in log-profits and t-statistics of means are given in captions of the above subfigures.

profits for a given functional form, sets of parameters, and k are denoted π^U_k.

Modified networks were then generated in which alterations of the unmodified networks were made in order to explore the effects of (i) weighted relationships (links of different weights), (ii) self-links (valuation independent of neighbors' actions (”stand-alone value”)), and (iii) clustering (higher frequency of links within groups of individuals). In order to keep the modified and unmodified networks otherwise comparable, the expected sum of degrees were held constant in the two classes of networks.

A constant-elasticity functional form, $v(q) = q^\alpha$, was given and valuation function of an individual of degree d given by $\theta(d) = 1 + \sqrt{d}$. For each generated network, α and c where drawn from a uniform distribution over $[0, 1]$, and f was then set to solve the equation $\pi^U_{k=1}(f) = \pi^U_{k=2}(f)$. I.e., I only consider situations in which a monopolist is indifferent between serving degree-1 consumers and not.[22] The goal of this simulation exercise is to estimate the expected difference in profits of the contracts $k = 1, 2$ in the ”modified” networks: $\pi^M_{k=2} - \pi^M_{k=1}$. If this difference is positive (negative) and statistically significant, I conclude that a certain alteration to the network makes it more (less) likely that the lowest contract is increased (decreased). A limitation of this approach is that the conclusions only are applicable to the specific network and parameter generating processes the simulations are based on, and the direction of the results may differ with other network generating processes as benchmarks. Summaries of the simulation results are provided in Figure A1.

[22]For each case, 4000 networks and a random set of parameters where generated. Simulation outcomes in which the compiler failed to solve for f or where profits were negative, $\pi^U_{k=1} = \pi^U_{k=2} < 0$, were omitted. These outcomes account for an excess of 1/4 of all outcomes.

A.3.1 Weighted links

Arguably, individuals value some relationships more than others. In this section, I modify the network to allow for different weighting of links between individuals in the sense that a consumer's value of a given quantity depends not only on *how many* neighbors who participate, but may also depend on *which* of his neighbors who participate.

In order to work the extension into the existing model with discrete types, I apply a mean-preserving spread approach. For each existing link, $g_{ij} = g_{ji} = 1 \in \boldsymbol{G}$ in the umodified network, let each $g_{ij} = g_{ji}$ take values $\{0, 1, 2\}$ with probabilities $(r/2, 1-r, r/2)$, where $r \in [0, 1]$. Thus, with probability r, a link is either severed or "doubled" with equal probability, or remains at $g_{ij} = g_{ji} = 1$ with probability $1 - r$. Thus, an individuals' network degree, $\tilde{d}_i(\boldsymbol{G}) = \sum_{j=1}^{n} g_{ij}$, remains the same in expectation. The question of interest is then which direction the monopolist's optimal cut-off of k is likely to go when some relationships are stronger than others.

There are two opposing effects: Some links having double weight means that the nodes at the end of these links are less dependent on other direct and indirect relationships to have their degree being at least k so that their participation constraints are satisfied. I.e., this should weaken the market size effect for the involved nodes, as well as the distribution effect of those in the neighborhood of nodes endowed with double links. Thus, this effect goes in the direction of increasing the lowest contract, k. On the other hand, as some links now are severed, nodes who were in C_k in the unmodified network may for the same k have a degree $d < k$ in the weighted network. For instance, at $k = 2$, a node who previously was located in a cycle, may now be out if the cycle is broken. In return, nodes connected to these nodes will have their degrees reduced if the lowest contract is increased. This effect strenghtens the market size and distribution effects and therefore goes in the direction of decreasing the lowest contract.

2867 simulations were conducted in which an existing link in the unmodified network got weight $g_{ij} = g_{ji} = 2$ (doubled) with probability .25 and severed, $g_{ij} = g_{ji} = 0$, with the same probability. Differences in log-profits in the two regimes for the weighted networks, $\pi^W_{k=2} - \pi^W_{k=1}$, are shown in Figure A1 (a). As illustrated, the results are noisy as there are several observations with both positive and negative values. However, the mean is positive and equal to 0.25 with a corresponding t-value of 29.9 under null hypothesis of zero mean. The result suggests that, in expectation, the optimal k is higher with a weighted network compared to an unweighted one under this specific network generating process. A plausible explanation is that weighted links increases the correlation of degrees between connected consumers. Thus,the nodes who have links with weight 2 are now less dependent on the participation of lower value types through direct and indirect connections. Thus, the participation effect and distribution effect from low-demand types to high-demand types are weakened, which incentivizes the seller to increase k.

A.3.2 Self links: Network independent types

Positive and negative self links

Two individuals with the same amount of participating neighbors may still value a product differently. In the same manner as in the extension with weighted links I apply a mean-preserving spread approach to types, where the valuation, θ of a given node i with degree d is given by $\theta_{id} = \theta(d + \hat{v})$, where $\hat{v}$ is a stochastic variable that take values $\{-1, 0, 1\}$ with probabilites $(\rho/2, 1 - \rho, \rho/2)$, respectively. $\theta(\hat{v})$ can therefore be seen as a stand-alone value of the good.

2910 simulations with $\rho = 0.25$ where conducted. Differences in log-profits under the two regimes with self links, $\pi^S_{k=2} - \pi^S_{k=1}$, are illustrated in Figure A1 (b). As in the weighted links case there is large variation. However, the mean difference in log-profits is -0.06 with t-value of -7.2. This suggests that with self links and keeping the expected degree constant, the optimal contract is in expectation lower than under a network without self links in this specific network generating process.

Why do we find the opposite result from the weighted links case? One explanation may be that weighted links is similar to adding self links on a pair of connected nodes. Thus, relative to the unweighted network, there will be an increased correlation in the valuation between connected nodes. Thus, if pairs or nodes either become weaker or stronger, the stronger pairs will be less affected by increasing the lowest contract. With self links there is no increased correlation in types among connected consumers, and an effect similar to the negative effect from the weighted link simulation dominates.

Positive self links

With both positive and negative self links, there are opposing effects as some consumers become less dependent on neighbors' actions (positive self links) while others become more dependent (negative self links) in order to adopt the product.

The following methodology was used to analyze the effect on having positive stand-alone values on the optimal participation: From the unmodified network a (positive) self link was added with probability .25, while an existing link between two consumers i and j, $i \neq j$ was severed with probability .125 keeping the expected sum of degrees constant across the modified and unmodified networks.

Results from 2926 simulated experiments are illustrated in Figure A1 (c). The mean difference in log-profits is 0.21 and significantly positive with a t-statistic of 27.1. However, as in the previous cases, there is large variation with a large share of the observations on the negative side.

A.3.3 Clustering

Empirical evidence from real world social networks suggests that two individuals are more likely to be friends if they share a friend. One common measure of such connections is *clustering* in which in some form counts the number of *triads* in the network, i.e., instances in which all nodes in group of three are linked to each other. The most basic measure is *overall clustering coefficient* which counts the share of instances where two nodes who share a connection also are connected to each other:

$$Cl^{overall}(\boldsymbol{G}) = \frac{\sum_{i;j\neq i;k\neq j;k\neq i} g_{ij}g_{ik}g_{jk}}{\sum_{i;j\neq i;k\neq j;k\neq i} g_{ij}g_{ik}}$$

There are also other measures of clustering (see e.g. Jackson (2008)), and the preferred measure depends on the phenomenon one wishes to study. For the current exercise, any commonly used clustering measure will have an increase in its coefficient.

In this exercise, clustering of a given random network was increased in the following manner. For each random network, then for each "star" in the network (where i is connected to j and k without j and k being connected), j and k were connected with probability .25, increasing the number of triads in the network. For each link added, a randomly selected link in the network was removed, keeping the number of links constant.

Results from 2776 simulations are illustrated in Figure A1 (d). The mean log-difference of $\pi^{C}_{k=2} - \pi^{C}_{k=2}$ is 0.29 and significantly positive with a t-statistic of 38. In addition to having a larger mean than the other cases, the distribution is more positively skewed. This suggest that it is more "representative" to exclude low degree consumers in more clustered networks, all else being equal. An explanation for this, similarly to the case of weighted links, is that the correlation in degrees between connected links increases relative to the unmodified network (high degree nodes are connected to high degree nodes). A larger share of nodes are likely to be in a C_2 (all nodes in a triad is in a cycle and therefore also a C_2), and a random node in a C_2 will also be less likely to be connected to a node outside C_2. Thus, the market size and distribution effects generated from low-demand to high-demand nodes are on average weakened with increased clustering.

B Empirical application: Selling mobile plans in Indian villages

The following example is based on social network data from Banerjee et al. (2013) initially used to analyze participation in microfinance loans in several villages in the Indian state of Karnataka. Based on questionnaires in 75 villages, 12 adjacency matrices were constructed for each village describing various types of social relations across villagers.[23] The analysis here is based on the so-called "friends" networks.[24] The first column in Table B1 provides descriptive statistics of the degree distribution across all of the 75 villages (degree 0 nodes omitted).[25]

[23] Some examples of relations are friendships, visits, advice, borrowing/lending money/kerosene, and temple/church/mosque companions.

[24] Friendships are undirected and unweighted and are based on mentioning or being mentioned to answering: "Name the 4 non-relatives whom you speak to the most." If i mentions j, j mentions i, or both, then i and j are defined as friends.

[25] The social network data were collected from a random sample of villagers, yielding a sample of approximately 46% of all households per village (Banerjee et al., 2013).

	$k=1$	$k=2$	$k=3$
Mean degree	2.64	3.18	3.93
Mode	1	2	3
Median	2	3	3
Standard deviation	1.82	1.65	1.45
Skewness (3rd moment)	2.76	3.38	3.51
Minimum ($=k$)	1	2	3
Maximum	39	34	22
Sum of degrees	39082	28054	4544
No. of buyers	14784	8828	1156
Share of total population	100%	59.7%	7.8%
Buyers with $d=k$	4323	3851	603
Buyers with $d \geq k+1$ and $\notin C_{k+1}$	1633	3821	553
- % of buyers with $d \geq k+1$	15.6%	76.8%	100%
Mean degree conditional on $d \geq k+1$	3.32	4.09	4.95

Table B1: Descriptive statistics and realizations of maximal equilibria in threshold games of complements in 75 villages in Karnataka, India. Data from "friend networks" in Banerjee et al. (2013). 2211 nodes without links are omitted.

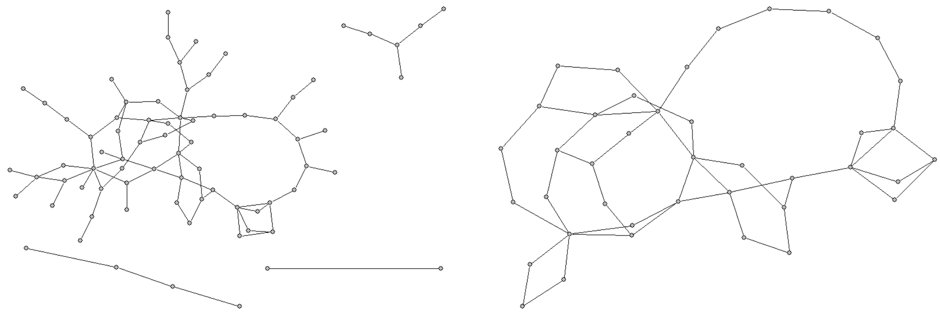
Consider a mobile phone operator offering a menu of vertically differentiated mobile plans to the villagers, and suppose a villager's utility of a minute of talk time is increasing in the number of friends subscribing to a mobile plan. Thus, under incentive compatible contracts, individuals with more friends subscribe to a plan with more talk time.

Suppose the phone operator knows the network structure and has estimated demand functions for each degree. The operator must then consider its choice of k, i.e., the lowest degree being willing to subscribe to a plan.

With full participation 14784 consumers subscribe to a plan. The most sold contract (4423 or 29.2%) is to consumers who only uses the phone to call one friend, and it is therefore not unimaginable that the monopolist incurs losses from these individuals through low prices and high information rents to more highly connected consumers.

If the operator should choose not to offer a contract to degree-1 consumers, it must also take into account the negative impact exclusion has on profits through decreased network benefits. As the second-to-last element in the first column in Table B1 reveals, not offering a contract to degree 1 nodes leads to an additional exclusion of 15.6% of the total population having degrees of at least 2 under full participation (market size effect). Figure B1 illustrates the set of participating nodes under the two regimes in one specific village.

Increasing the lowest contract further, i.e., not offering a contract to degree-2 nodes appear more costly in terms of participation and degree distribution. Now 76.8% of consumers having degrees of 3 or more when $k=2$ are excluded from the market (nodes in C_2, but no in C_3). In fact, in 37 of the 75 villages *no one* subscribe to the plan at $k=3$. Of all the 14784 villagers, no one is in a C_4, thus $k=3$ is the highest lowest contract which is feasible with a positive level of participation.



(a) 75 participating nodes with $k = 1$ **(b)** 40 participating nodes with $k = 2$

Figure B1: Friend network from "Village 10" in Karnataka from Banerjee et al. (2013). "Name the 4 non-relatives whom you speak to the most." There are no nodes in C_3.

In addition to a substantial negative market size effect from excluding low-degree consumers, the table also captures the distribution effect. The maximum degree decreases in k, and the mean degree is substantially lower than the mean without exclusion conditional on degrees strictly higher than k (e.g. the mean degree conditional on having a degree of at least 2, decreases from 3.32 to 3.18 when k is increased from 1 to 2). Generally the skewness increases in k, suggesting that a larger mass are located to the left in the degree distribution.[26]

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[26] Skewness is not a perfect measure since decreasing standard errors also partially explain increased skewness.

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