Equilibrium Coalitional Behavior*

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Abstract

I develop a solution concept, equilibrium coalitional behavior (ECB), which captures foresight and imposes the requirement that each coalition in a sequence of coalitional moves chooses optimally among all its available options. The model does not require, but may use, the apparatus of a dynamic process or a protocol that specifies the negotiation procedure underlying coalition formation. Therefore, it forms a bridge between the noncooperative and the cooperative approaches to foresight. ECB refines subgame perfect equilibrium in extensive form games of perfect information and provides a complete characterization of the core in characteristic function games. Through applications it is shown that ECB provides a unified approach to study a wide range of problems involving sequential coalitional actions, which have hitherto been solved on a case-by-case basis.

JEL classification: C70; C71; C72; D71

1 Introduction

One perspective on the challenges of game theory in general and cooperative game theory in particular is given by Harsanyi and Selten:

> An even more serious shortcoming of classical game theory is its failure to provide any usable solution concepts for some theoretically and empirically very important classes of cooperative (and of less than fully cooperative) games. These include:
>
> 1. Games *intermediate* between fully cooperative and fully noncooperative games. Examples are games where some types of agreements are enforcable while others are not; games where some groups of players are able to make enforcable agreements but others are not; and games where enforcable agreements can be concluded at some stages of the game but not at other stages.
> 2. Cooperative games with a *sequential* structure. (There is some overlap between cases 1 and 2.) These are games involving two or more successive stages and permitting agreements to be built up gradually in several consecutive steps. **(Harsanyi and Selten (1988))**

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Many economic interactions are sequential. Even when such interactions involve agreements among coalitions, the literature leans toward noncooperative game theory. This is because there is no generally accepted way of solving these problems using a cooperative solution concept, i.e. without explicitly specifying the bargaining/negotiation procedures. But applying a noncooperative approach has its own problems, in particular such an approach may require ad hoc assumptions on how players negotiate and it comes at the cost of greater complexity. This has been eloquently expressed by Aumann:

> when one does build negotiation and enforcement procedures explicitly into the model, then the results of a noncooperative analysis depend very strongly on the precise form of the procedures. (...) But problems of negotiation are usually more amorphous; it is difficult to pin down just what the procedures are. More fundamentally, there is a feeling that procedures are not really all that relevant (...) Finally, detail distracts attention from essentials. Some things are seen better from a distance; the Roman camps around Metzada are indiscernible when one is in them, but easily visible from the top of the mountain. **Aumann (1987)**

This is the reason why one might not want to stick with noncooperative game theory in sequential games with coalitional actions. Although cooperative game theory has the advantage of abstracting away from the details, it is not suited to deal with sequential problems. The reason is that most solution concepts in cooperative game theory are static. For instance take the core, which is the set of outcomes that cannot be improved upon by any group of players through an action that the group has the power to implement. But, the definition of the core does not address what happens after the status quo is changed; it does not consider the possibility that the initial objection might be followed by further objections.

This is also an issue when one tries to incorporate foresight into cooperative solution concepts. Consider the notion of the core again, even if we imagine that a move might be followed by other moves it is not obvious how to extend the definition to this case. Intuitively, this would require considerations similar to backward induction to be introduced in a cooperative setting. But, in the absence of the structure of an extensive form, it is not at all obvious how to incorporate such considerations.

That is why, although a number of solution concepts have been developed to incorporate foresight in the cooperative approach, it has been hard to incorporate the idea of what Ray and Vohra (2014) calls 'maximality' to these solution concepts. Which simply refers to the observation that in these solution concepts coalitions may not take the optimal moves that are available to them.

This issue resembles the problem that in sequential move noncooperative games, a Nash equilibrium may not prescribe the optimal course of action at 'unreached' nodes of an extensive form. This has been famously resolved by subgame perfect equilibrium by requiring optimality at every place where a decision is made. Although subgame perfection and backward induction cannot be readily incorporated into a cooperative domain, it is at the very least desirable that in the specific problems in which such an approach is applicable it should be prescribed.

In this paper my main goal is to come up with a solution concept that incorporates foresight in games with coalitional actions and that (*i*) prescribes the optimal course of action at any place where a decision is made, just like the noncooperative approach and (*ii*) abstracts away from the details of the negotiation and enforcement procedures, just like the cooperative approach.

The main contributions of coming up with the solution concept, *equilibrium coalitional behavior* (ECB), are:

1. *Linking the noncooperative and the cooperative approaches to farsighted coalition formation*

   ECB does not require, but might use, the details of how players negotiate to form coalitions. Therefore, it is applicable to both noncooperative and cooperative sequential move games and it can be used to study foresight in a cooperative setting. This means that ECB has the potential to form interesting relationships between very different solution concepts. The relationships ECB engenders range from a fully noncooperative dynamic solution concept, the subgame perfect equilibrium, to the static cooperative solution concept of the core, to the more recently developed solution concepts of Konishi and Ray (2003) and Dutta and Vohra (2015).

2. *Providing a unified approach to study sequential games with coalitional actions*

   A wide range of situations involve sequential actions by groups of players; examples include club/political party formation, conflict, customs unions, network formation and legislation. Although in most of these situations there is no explicit negotiation procedure, the literature leans toward noncooperative game theory to study these problems. This means that the description of the game must be augmented with an (oftentimes) arbitrary negotiation procedure. The lack of consistency in the literature means that each problem is handled in a case-by-case basis and depending on the situation different solution concepts and different negotiation procedures are used. ECB provides a unified and consistent way to study these problems, which will be shown in Section 7 through the characterization of ECBs in two general classes of games that have been previously used in the literature to study a range of situations.

I will use the following example due to Roberts (2015) to demonstrate the solution concept and these points.

Example 1. *Roberts (2015)*
There is a society $N = \{1, 2, ..., n\}$ and an initial club $F = \{1, 2, ..., f\}$ where f is a positive integer. At the initial period, any strict majority within the club can choose to expand or shrink the club at a cost. Once the club changes any strict majority within the new club can choose to expand or shrink this new club. This can go on indefinitely. There is a seniority system such that only clubs of the form $s_j = \{1, 2, ..., j\}$ for some integer j are allowed. Everybody in the society wants to be a part of the club and conditional on being in the club, everybody prefers a smaller club to a larger club. Players discount the future with a discount factor arbitrarily close to 1.

Coalitions are the main actors in this problem, which means that we cannnot study it using a noncooperative solution concept without including details on how players negotiate to take an action that can only be taken by a coalition. The details will inevitably be arbitrary.

Roberts (2015) solves the problem with two different solution concepts, Markov voting equilibrium and median voter rule. Acemoglu, Egorov and Sonin (2012) also solve this problem with two different approaches. They define a noncooperative game by introducing a protocol and they study the Markov perfect equilibria of the resulting game. They also take an axiomatic approach to show that the result does not depend on the details introduced in the noncooperative approach. These two papers alone that apply four different solution concepts to similar problems show how difficult it is to handle sequential move coalitional games in a systematic manner.[1]

ECB can be directly applied to the problem without any need of introducing details on how agreements are reached and it provides a simple and intuitive solution. To see how, we first need to see the domain of ECB, the extended coalitional game.

An extended coalitional game includes a set of players, a set nodes, a set of actions available at each node to each coalition and the utilities defined over action sequences. For instance, in the game above the set of players is $\{1, 2, ..., n\}$, each node corresponds to a possible club and the actions at each node correspond to the decision of a majority within the current club to change the club. For example, if we are at a node corresponding to the club $\{1, 2, 3\}$ then the coalition $\{1, 2\}$ (as well as $\{2, 3\}$ or $\{1, 3\}$ or $\{1, 2, 3\}$) will have the possible action of expanding the club by admitting 4 or throwing 3 out of the club.[2] Finally, the utility of an action sequence would be the discounted utility of the clubs visited along the sequence minus the transaction costs.

An extended coalitional game is a generalization of the extensive form under perfect information, where the three main differences are: (i) Coalitions might take actions, (ii) there might be multiple individuals or coalitions capable of taking actions at a certain node and (iii) The game need not be a game that is representable as a tree, for instance the representation might include cycles.

A coalitional behavior assigns a single action to each node of an extended coalitional game. The use of a coalitional behavior imposes consistent expectations on the side of individuals and coalitions. An ECB is simply a coalitional behavior that satisfies sequential rationality for coalitions, i.e. that is immune to coalitional deviations at each node of the game. Hence, under an ECB each coalition is acting optimally given its expectation (the coalitional behavior).

Although the precise definition of a coalitional deviation will be provided in Section 3, this information is enough to see how ECB solves Example 1. If for any reason the club that is composed of only player 1 forms then this club will never change as this club is 1's favorite and he is the only member of the club. Now consider the club that is composed of players 1 and 2, any strict majority should include player 2 and since the club is player 2's favorite, this club should also be stable. But given that this club is stable the club composed of players $1, 2$ and 3 cannot be stable, as coalition $\{1, 2\}$ forms a majority and they prefer the club composed of 1 and 2 to the one composed of $1, 2$ and 3. But then the club composed of players $1, 2, 3$ and 4 is also stable as any majority should include players 3 or 4 and there is no stable smaller club that includes these players. The argument can continue and by induction we can show that $s_j = \{1, 2, ..., j\}$ is stable iff $j = 2^k$ for some $k = 0, 1, 2, ..$

The argument is directly applied to the description of the game without any need of introducing details on how agreements are reached. In Section 7, I will characterize the ECBs of two general classes of games, which include this problem and various other applications. The section establishes ECB as a unified approach to solve problems with sequential coalitional actions where there is no obvious way to apply noncooperative

[1] It is important to note that both of the papers study a more general class of problems, nevertheless ECB can be applied to the more general class they study as we will see in Section 7.

[2] More precisely, if we are at a node corresponding to the club $\{1, 2, .., K\}$ then any strict majority will have the option of admitting $\{K + 1, .., s\}$, where $K < s \leq n$ or throwing $\{s, .., K\}$ out of the club, where $1 < s \leq K$.

solution concepts.

Although this example shows the advantage of using ECB over noncooperative solution concepts, it does not explain the problem of maximality of cooperative solution concepts that incorporate foresight. Most of the cooperative solution concepts that incorporate foresight, such as the *farsighted stable set*[3] makes the same prediction as ECB in this example, however they suffer from the issue of maximality that noncooperative solution concepts have no problem with. This can be easily seen in a simple two period noncooperative game.

Example 2. *A Noncooperative Game*

There are four states, A, B, C and D. A is the status quo and player 1 can choose to change the state to B or not. If player 1 changes the state to B, then player 2 can change it to C or D or he might choose not to change the state. Utilities are defined over states and given by $u(A) = (1, 1)$, $u(B) = (0, 0)$, $u(C) = (0, 4)$ and $u(D) = (2, 2)$, where the first entry denotes the payoff of player 1.

In this example, ECB can be found through backward induction. Therefore, it is equivalent to the unique subgame perfect equilibrium, which states that 1 will not change the state at A and 2 will change the state to C at B.

This is not the case for the *farsighted stable set*. Under this solution concept player 1 would choose to move to state B, because she 'unreasonably' believes that player 2 would choose to move to D instead of C when he gets to make a decision. But once we are at state B player 2 has no incentive to move to state D, as C provides a strictly higher payoff to him. In a nutshell this is the problem of 'maximality', that players form unreasonable expectations and take suboptimal actions based on these expectations.

The problem of maximality is not limited to the farsighted stable set nor is it limited to this specific example. It has proven hard to incorporate such considerations into cooperative solution concepts, mostly because (unlike this example) the domain they are defined on does not admit an obvious way to incorporate considerations like backward induction. For a more general discussion see Ray and Vohra (2014). It will soon become clear that by defining the concept as a best response to a certain arrangement, ECB avoids these problems and it allows for backward induction when possible.

As we have already seen, ECB can be applied to both cooperative games in which the main building block is coalitional actions and to noncooperative games. Through its relationship to attractive solution concepts in different domains ECB provides a link between the noncooperative and the cooperative approaches to foresight. ECB refines subgame perfect equilibrium in extensive form games of perfect information (see Section 4), provides a complete characterization of the core in characteristic function games (see Section 5) and it is closely related to cooperative approaches to foresight developed by Konishi and Ray (2003) and Dutta and Vohra (2015) (see Section 6).

Finally, there are many situations in the literature that involve coalitions sequentially building up agreements and which have been traditionally analyzed within the context of noncooperative games, in particular with the use of subgame perfect equilibrium and its refinements. These situations include network formation (Aumann and Myerson (1988)), sequential formation of binding agreements (Bloch (1996), Ray and Vohra (1999)), dynamic club formation (Barbera, Maschler and Shalev (2001), Roberts (2015)) and various political games (Acemoglu, Egorov and Sonin (2008,2012)). To study these situations in

[3] Formal definition of the farsighted stable set will be given in Section 6, but no knowledge of the concept is needed to understand the following argument.

a noncooperative setting, the authors needed to complement the description of the game with the rules of coalition formation and in most cases they also needed to refine subgame perfect equilibrium to get reasonable predictions.

This inevitably brings a level of arbitrariness to the analysis. ECB provides a simpler and consistent way to approach these problems without specifying an arbitrary negotiation procedure and without using a wide range of different solution concepts. This will be shown through the characterization of ECBs in two classes of games that can be used to study the situations analyzed in the works cited above.

I will start with the Literature Review in Section 2. In Section 3, I define the domain and the solution concept. In Sections 4, 5 and 6, I study extensive form games of perfect information, characteristic function games and abstract games, respectively. In Section 7, through applications I show that ECB provides a unified approach to study sequential games with coalitional actions. Section 8 concludes by discussing some issues regarding ECB.

2 Literature Review

The recent literature on coalition formation has sought to incorporate foresight in coalitional decision making. Freely borrowing from non-cooperative concepts and tools, one strand of the literature has modeled coalition formation process explicitly as a non-cooperative game. This is done at the expense of complexity and sometimes unintuitive assumptions on the negotiation procedure among the individuals.

Another strand of the literature abstracts away from the details of the negotiation procedure and takes coalitional actions as opposed to individual actions as the main building block of the model. This is the cooperative approach and it can be roughly divided into two: the static and the dynamic approach. The latter differs from the former by explicitly modeling the coalition formation process as a dynamic process.

In this section I will briefly review these strands of the literature and argue that ECB is the natural result of the developments and it stands at the intersection of all three approaches. Inevitably the review is incomplete, for extensive reviews see Mariotti and Xue (2003), Ray (2008) and Ray and Vohra (2014).

2.1 The Noncooperative Approach

The noncooperative approach is by far the most popular approach to analyze sequential games of coalition formation. This is in part due to the cooperative approach's failure to incorporate 'maximality' of actions in its solution concepts (see Ray and Vohra (2014)).

In this approach, typically the negotiation process is modeled as offers and counteroffers and subgame perfect equilibrium and its refinements are used to solve the problem. The models heavily rely on the details of coalition formation which might make the solution sensitive to the details of the game and can make the game complex to solve.

Section 7 contains examples of papers that use the noncooperative approach and shows how ECB can be applied to the situations they analyze without the assumptions on how individuals interact and form coalitions. This literature is vast, but apart from the ones I mention in Section 7, prominent examples include Rubinstein (1982), Stahl (1977), Chatterjee et al. (1993), Okada (1996), Selten (1981), Krishna and Serrano (1996) and Moldovanu and Winter (1995).

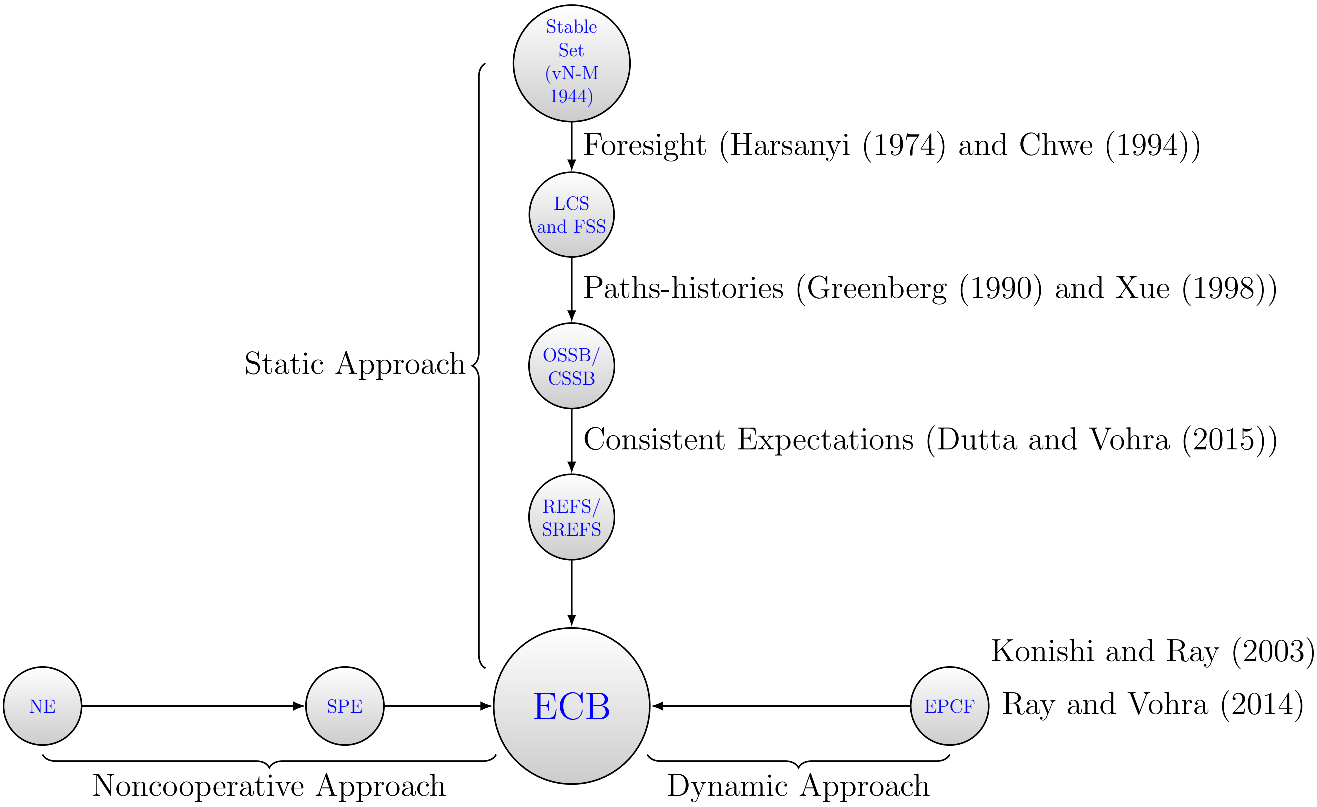


Figure 1: ECB is where the noncooperative, static and dynamic approaches meet[4]

Any perfect information extensive form game is an extended coalitional game and hence ECB can be directly applied to these games. Furthermore given the close relation ECB has to subgame perfect equilibrium (see Section 4), ECB can be seen as the natural extension of subgame perfect equilibrium to sequential games with coalitional actions.

2.2 The Static Approach

The quest to incorporate foresight into the static cooperative solution concepts goes back at least to Harsanyi (1974), who criticized von Neumann and Morgenstern's stable set (1944) for being myopic. Chwe (1994) formalized Harsanyi's criticism and developed the solution concepts of the farsighted stable set and the largest consistent set. These are set valued concepts in the tradition of von Neumann and Morgenstern's stable set, which use the indirect dominance relation instead of the direct dominance relation the stable set uses.[5]

Xue (1998) argued that this approach is not entirely satisfactory as farsighted players should not only consider the final states their actions lead to, but they should also consider how these states are reached. By using Greenberg (1990)'s framework Xue (1998) proposed to use paths to incorporate foresight into his solution concepts. But Xue (1998)

[4]NE and SPE correspond to Nash equilibrium and subgame perfect equilibrium, respectively. All other acronyms correspond to the solution concepts of the authors mentioned on the arcs.

[5]For more on farsighted stable set see Diamantoudi and Xue (2003), Mauleon, Vannetelbosch and Vertoge (2011) and Ray and Vohra (2015). For more on the largest consistent set see Beal, Durieu and Solal (2008), Bhattacharya (2005), Herings, Mauleon and Vannetelbosch (2009), Mauleon and Vannetelbosch (2004), Page, Wooders and Kamat (2005) and Xue (1997).

still used a framework in which players form arbitrary expectations based on optimism or pessimism to evaluate different sets of paths. Questions remain about these extreme expectations players hold to evaluate sets of paths, for details see Bhattacharya and Ziad (2012), Herings, Mauleon and Vannetelbosch (2004) and Ray and Vohra (2014).

Dutta and Vohra (2015) propose to deal with these issues by embodying the farsighted stable set with consistent expectations and introducing one-step deviations (also see Jordan (2006) for an earlier work concerning common expectations and farsighted stability).

The trend in the static approach is apparent, at each step the solution concepts get one step closer to the noncooperative approach. It started by incorporating foresight, then paths and histories and then consistent expectations and one-step deviations (see Figure 1). ECB, which can be cast as a solution concept in the static approach (see Section 6.1) completes this evolution by formalizing the logic of subgame perfection in sequential coalitional games. By doing so it avoids the drawbacks of the earlier approaches.

2.3 The Dynamic Approach

Given the problems of the static approach some authors found the way out in introducing an explicitly dynamic solution concept:

> there are limits to how effectively one can capture farsightedness in a static concept of stability. (...) in the present context it seems too confining not to introduce some details (as well as explicit dynamics). **(Ray and Vohra (2014))**

The main solution concept in the dynamic approach is the EPCF (Konishi and Ray (2003) and Ray and Vohra (2014)), which models coalition formation as an explicitly dynamic process and the payoffs are discounted with a common discount factor. The approach proved to be useful in avoiding the pitfalls of the static approach and has also been used by Dutta, Ghosal and Ray (2004) and Vartiainen (2011). ECB, which can be cast as a solution concept in the dynamic approach is related to the EPCF. The relation will be discussed in Section 6.2.

2.4 Equilibrium Coalitional Behavior

ECB can be applied to noncooperative games and the domain of ECB allows us the flexibility to define it both as a static and a dynamic solution concept. Hence, unlike all of the other solution concepts discussed above ECB belongs to all of the three approaches. Furthermore, as will be shown, ECB is closely related to subgame perfect equilibrium in the noncooperative approach, Dutta and Vohra (2015)'s solution concept in the static approach and EPCF in the dynamic approach. Therefore, through linking all of the three approaches, ECB unifies the field used to study coalition formation games under foresight (see Figure 1).

3 Preliminaries

The following example will be used to illustrate the domain and the solution concept.

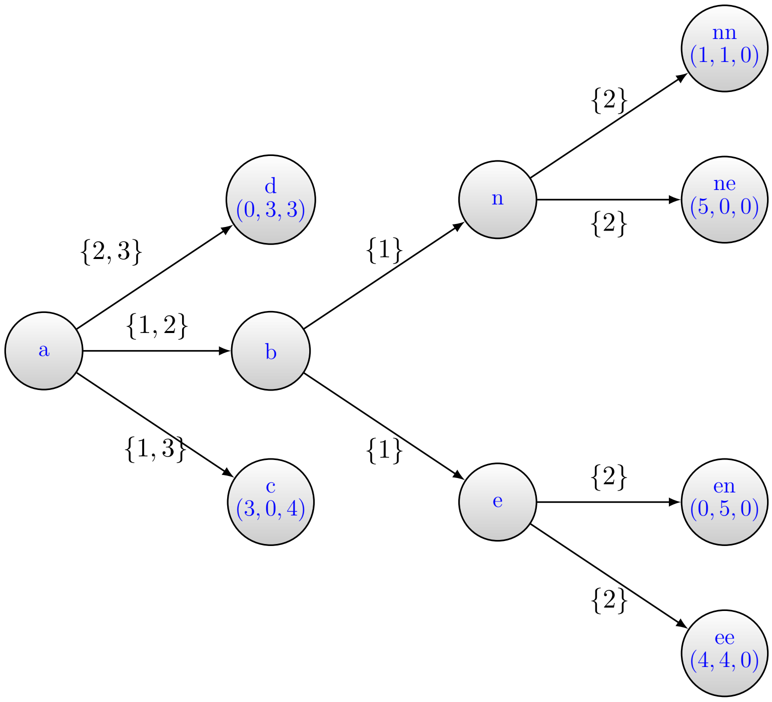


Figure 2: The Partnership Game

There are three players, 1, 2 and 3. Any two of these three players can form a binding partnership. When two players form a partnership the remaining player gets a payoff of 0. If player 1 forms a partnership with player 3 then they get a payoff of 3 and 4, respectively. If player 2 forms a partnership with player 3 then they get a payoff of 3 and 3, respectively. If players 1 and 2 form a partnership then first player 1 chooses whether to exert effort or not and then observing player 1's choice, player 2 chooses whether to exert effort or not. The game ends after player 2 makes his decision and the payoffs are realized.

This game can be represented with a tree, where at the start of the game any coalition of size 2 can form a partnership. If a coalition that includes player 3 forms then the game ends, otherwise players 1 and 2 play a sequential noncooperative game in which they sequentially choose whether to exert effort. Both the game and the payoffs are represented in Figure 2.

This simple game illustrates the points made by Harsanyi and Selten (1988):

1. It is a game *intermediate* between fully cooperative and fully noncooperative games. In particular the game is clearly divided into a cooperative stage in which two players form a binding partnership and a noncooperative stage in which players individually choose their effort levels. The partnership decision is enforcable and binding, whereas the effort decisions are made individually and binding agreements on the effort decisions cannot be made.

2. The game has an obvious *sequential* structure. The partnership decision is followed by the effort game.

3. Noncooperative solution concepts, such as the subgame perfect equilibrium, cannot be applied to the game as it is. Such solution concepts require further information. We have to make assumptions on how groups decide on certain actions, such as

the decision to form a partnership. These assumptions will inevitably be arbitrary and the solution may depend on the particular assumptions made by the modeler. Furthermore they come at the cost of greater complexity.

The game can be complicated in many directions; it might have multiple stages in which coalitions take actions, it might have stages at which only coalitions with certain properties might take actions, it might have an infinite time horizon. Examples of more complicated games having these features will be discussed throughout the paper.

3.1 The Extended Coalitional Game

The domain of ECB is the extended coalitional game. An extended coalitional game is defined as $\Gamma = \{N, Z, \{A_z\}_{z\in Z}, \{\succeq_i\}_{i\in N}\}$, where N is the set of players, Z is the set of nodes, A_z is the set of actions available at node $z \in Z$ and $\succeq_i$ is the preference relation of player $i \in N$ over the set of terminal paths, which will be defined shortly.

An action is a triple (z, z', S), where the first entry $z \in Z$ denotes the node at which the action can be taken, the second entry $z' \in Z$ denotes the node to which the action is leading to and the third entry denotes the coalition $S \subseteq N$ that can take the action. This coalition is called the *initiator* of the action.

For reasons that will soon become clear, I represent taking 'no action' as a particular action denoted by $(z, z, \emptyset)$. For any action (z, z', S), I require $S \neq \emptyset$ iff $z' \neq z$. A_z denotes the set of all actions that can be taken at node $z \in Z$, hence for all $(x, y, S) \in A_z$ we have $x = z$. Furthermore, I require that $A_z \neq \emptyset$. This does not mean that an action should be taken at z; in particular if no action is available at z then $A_z = \{(z, z, \emptyset)\}$.

A path is a sequence of actions $\{a_k\}_{k=1,\ldots,K} = \{(z_k, z_{k+1}, S_k)\}_{k=1,\ldots,K}$, where K might be infinite. For any path $\{a_k\}_{k=1,\ldots,K}$, if $a_i = (z, z, \emptyset)$ for some $i \in \{1, .., K\}$ then $i = K$. This simply states that the action of taking no action cannot be repeated. A path is terminal if it is infinite or if $a_K = \{(z, z, \emptyset)\}$ for some $z \in Z$. Let H denote the set of all terminal paths. $\succeq_i$ denotes the preference relation of $i \in N$ on H. The following examples illustrate the domain.

Example 3. *The Partnership Game*

This game is defined above. There are three players $N = \{1, 2, 3\}$. There are 10 nodes, $Z = \{a, b, c, d, n, e, ne, nn, en, ee\}$. At node a any two player coalition can choose to form a partnership, i.e. $A_a = \{(a, b, \{1, 2\}), (a, c, \{1, 3\}), (a, d, \{2, 3\})\}$. At node b, player 1 can choose to exert effort or not, i.e. we have $A_b = \{(b, n, \{1\}), (b, e, \{1\})\}$. The actions at nodes n and e are defined similarly. All other nodes are terminal nodes, i.e. $A_j = \{(j, j, \emptyset)\}$ for $j = c, d, nn, ne, ee, en$. Note that every terminal path ends up at a terminal node and the terminal node at which the path terminates determines the utility of the path, which is represented on the graph.

Most of the solution concepts in the static and dynamic approaches (see the Literature Review) are defined on the abstract game (see Section 6). The abstract game takes the convention that the nodes correspond to the states, therefore deciding to stay in one state (node) is a possible action at each node of an abstract game. The same convention might also be used in an extended coalitional game. The following example demonstrates a game with no exogenously stable nodes.[6]

[6] An exogenously stable node is a node z such that $A_z = \{(z, z, \emptyset)\}$.

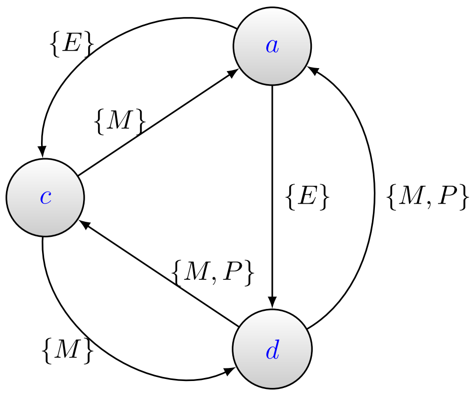


Figure 3: Regime Change

Example 4. *Regime Change*

This example is a slight modification of Example 1 *in Acemoglu, Egorov and Sonin (2012). There is a society composed of three groups, the elite (E), the middle class (M) and the poor (P). There are three states, absolutist monarchy (a), constitutional monarchy (c) and democracy (d). In absolutist monarchy, E decides on whether to change the regime; in constitutional monarchy M decides on whether to change the regime and in democracy M and P can together decide on whether to change the regime. Players have utilities defined on the states:* $u_E(c) > u_E(a) > u_E(d)$, $u_M(c) > u_M(d) > u_M(a)$ *and* $u_P(d) > u_P(c) > u_P(a)$ *and a discount factor* δ*. The utility of each terminal path is the discounted utility of states visited along the path. The game is represented in Figure 3.*

In this game $N = \{E, M, P\}$, $Z = \{a, c, d\}$*. At node* a*,* E *may choose to transition to* c *or* d *or do nothing, i.e.* $A_a = \{(a, c, E), (a, d, E), (a, a, \emptyset)\}$*. Similarly,* $A_c = \{(c, a, M), (c, d, M), (c, c, \emptyset)\}$ *and* $A_d = \{(d, a, \{M, P\}), (d, c, \{M, P\}), (d, d, \emptyset)\}$*. Finally, the utility of player* $i \in N$ *from the terminal path* $\{(z_k, z_{k+1}, S_k)\}_{k=1,2,...,K}$ *is*

$$\sum_{k=1,..,K} \delta^{k-1} u_i(z_k) + \sum_{k=K,...} \delta^k u_i(z_K)$$

Remark 1. An extended coalitional game might leave out many details such as the procedure with which players communicate to take actions and the order in which coalitions are allowed to move. This is the main strength of the domain and the solution concepts defined on this domain.

Remark 2. The game can be conveniently represented as a directed labeled graph $(Z, \{A_z\}_{z \in Z})$. Where Z is the set of nodes and A_z is the set of arcs whose tail is z and which is labeled with the corresponding initiators. See Figures 2 and 3 for the graphical representations of the Partnership Game and the Regime Change. This graphical representation includes every ingredient of the game except for the preferences. Hence, when convenient I am going to explain the game as a graph instead of writing down all the ingredients.

3.2 Equilibrium Coalitional Behavior

A coalitional behavior prescribes a unique action at each node of an extended coalitional game. Let A denote the set of all actions, i.e. $A = \cup_{z \in Z} A_z$.

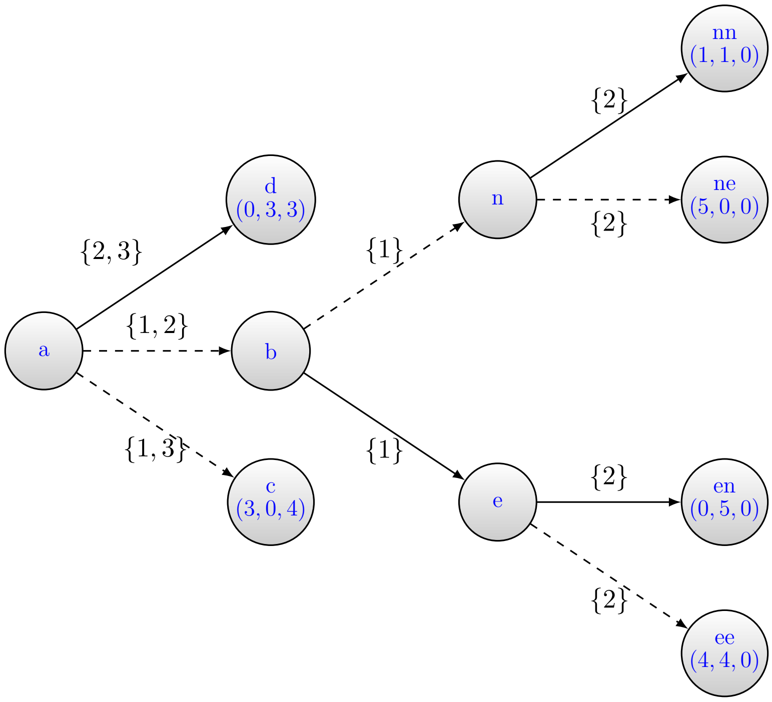


Figure 4: ϕ_1 defined for the Joint Project

Definition 1. *Coalitional Behavior*

A coalitional behavior is a mapping $\phi : Z \to A$ where $\phi(z) \in A_z$ for all $z \in Z$.

For example for the Partnership Game, let ϕ_1 be a coalitional behavior that specifies that at node a coalition $\{2,3\}$ will form, at node b player 1 will exert effort and player 2 will not exert effort in any node at which he takes an action. Then $\phi_1(a) = (a, d, \{2,3\})$, $\phi_1(b) = (b, e, \{1\})$, $\phi_1(n) = (n, nn, \{2\})$, $\phi_1(e) = (e, en, \{2\})$ and $\phi_1(j) = (j, j, \emptyset)$ for $j = c, d, nn, ne, en, ee$. This coalitional behavior is represented in Figure 4.

Note that each coalitional behavior defines a predicted terminal path at each one of the nodes, I will simply call this the path of play. For any $z \in Z$ let H_z denote the set of terminal paths that start at z.

Definition 2. *Path of play*

Given a coalitional behavior ϕ, a path of play is a mapping $\sigma : Z \to H$ such that for each $z \in Z$, $\sigma(z) = \{(z_k, z_{k+1}, S_k)\}_{k=1,..,K} = \{\phi(z_k)\}_{k=1,..,K} \in H_z$, where $z_1 = z$ and $z_{k+1} = \phi^2(z_k)$ for all $k = 1, ..., K$.[7]

For instance lets say that σ_1 is the path of play defined by ϕ_1 above. Then $\sigma_1(a) = \{(a, d, \{2,3\}), (d, d, \emptyset)\}$, $\sigma_1(b) = \{(b, e, \{1\}), (e, en, \{2\}), (en, en, \emptyset)\}$ and so on.

Now, we would like to put some restrictions on a coalitional behavior to make it a prescription that will be followed by rational individuals. Intuitively ϕ_1 does not satisfy such a criterion, since player 3 is better off forming a coalition with 1 instead of 2. Furthermore 1 also prefers this to ϕ_1. Therefore, we can say that the coalition $\{1,3\}$ has a profitable deviation from ϕ_1 in which player 3 refuses to form a partnership with 2 and instead 1 and 3 form a partnership. To formalize and generalize this idea I will first define a deviation from a coalitional behavior and then a profitable deviation.

[7] $\phi^2(z_k)$ denotes the second entry of $\phi(z_k)$.

The idea is that a coalition $S \subseteq N$ can deviate from a coalitional behavior ϕ to a coalitional behavior ϕ' if it has the power to induce ϕ' from ϕ by refusing to take actions specified by ϕ or by taking actions not specified by ϕ. Intuitively, S cannot refuse to take an action that is taken by a coalition T with $S \cap T = \emptyset$ and S cannot take an action that can only be taken by a coalition T with $T \not\subseteq S$.

Hence, a deviation by a coalition S from a coalitional behavior ϕ to a coalitional behavior ϕ' will only be feasible if S can induce ϕ' by refusing to take actions specified by ϕ whose initiators have a nonempty intersection with S and by taking actions not specified by ϕ whose initiators are a subset of S. Finally, we will say that a deviation is profitable if at every node at which an action changes the deviating coalition is better off at the new path of play.

Definition 3. *Coalitional Deviation*

$S \subseteq N$ has a deviation from a coalitional behavior ϕ to a coalitional behavior ϕ' if for every $z \in Z$ such that $\phi(z) \neq \phi'(z)$ we have

- *If $\phi(z) = (z, z', T)$ where $T \neq \emptyset$ then $S \cap T \neq \emptyset$ (If an action specified by ϕ is not taken, then S has a member who can refuse to take this action)*
- *If $\phi'(z) = (z, z', T)$ then $S \supseteq T$ (If an action not specified by ϕ is taken, then S should be able to induce this action)*

We say that the deviation by S is profitable if for every $z \in Z$ such that $\phi(z) \neq \phi'(z)$ we have $\sigma'(z) \succ_i \sigma(z)$ for all $i \in S$.[8]

Two aspects of the definition is worth noting: (i) a deviation might involve any number of actions, for instance the grand coalition can impose any coalitional behavior from any other coalitional behavior and (ii) unlike subgame perfect equilibrium, we say that a deviation is profitable if the coalition is better off at every point where an action changes. This is necessary for a definition involving coalitional deviations as opposed to a definition that only lets individuals deviate, as some subset of the coalition might have no incentive to keep their part of the deviation. This might be seen most clearly in a simple extensive form. For instance consider the noncooperative stage of the Partnership Game as a separate game.

In this game first player 1 chooses whether to exert effort and then observing player 1's choice player 2 chooses whether to exert effort. Suppose a coalitional behavior specifies that both players will choose to exert 'no effort' at each node. There is a coalitional deviation from this coalitional behavior by $\{1, 2\}$ in which player 1 chooses to exert effort at node b and player 2 chooses to exert effort at node e. This deviation increases the payoff of both 1 and 2.

But there is a problem with this deviation, in particular once player 1 chooses to exert effort, player 2 has no incentive to keep his part of the deviation. In other words in this deviation player 2 has to change his action at e, but this does not increase his payoff at the new path of play starting at e. The definition of a profitable deviation above takes this into account by requiring that the deviation increase the payoff of individuals at every node at which an action changes. Hence, in this example there is no profitable deviation from the coalitional behavior that specifies that every player will always choose to exert no effort.

[8] σ' and σ are the path of plays corresponding to ϕ' and ϕ, respectively.

If we get back to the Partnership Game, note that coalition $\{1,3\}$ has a profitable deviation from ϕ_1 in which player 3 refuses to form a partnership with 2 and instead players 1 and 3 form a partnership. The deviation changes the action at a only and at the new path of play both players 1 and 3 are better off, hence this is a profitable deviation by $\{1,3\}$. Lets call the resulting coalitional behavior ϕ_2, i.e. $\phi_2(a) = (a, c, \{1,3\})$ and $\phi_2(j) = \phi_1(j)$ for all $j \neq a$.

Now, player 1 has a profitable deviation from ϕ_2 in which he chooses not to exert effort at node b, the deviation changes the action from node b only and the new path of play is preferred by 1. Lets call the resulting coalitional behavior ϕ_3, i.e. $\phi_3(b) = (b, n, \{1\})$ and $\phi_3(j) = \phi_2(j)$ for all $j \neq e$.

There is a deviation from ϕ_3 by coalition $\{1,2\}$ in which 1 refuses to form a partnership with 3 but instead 1 and 2 form a partnership and both players 1 and 2 choose to exert effort when they are choosing between effort or no effort. Note that this deviation requires the deviating coalition to change its action at 3 nodes. Although this deviation increases the payoffs of everyone involved from the root of the game, it is not a profitable deviation because it does not increase the payoffs of every player involved at each node where an action changes, for instance at node e in which player 2 changes his action. Indeed, there is no profitable deviation from ϕ_3.

Now, we are ready for the definition of an ECB. It is simply a coalitional behavior from which there exists no profitable deviation.

Definition 4. *Equilibrium Coalitional Behavior (ECB)*
A coalitional behavior ϕ is an ECB if there does not exist a profitable coalitional deviation from ϕ.

In the Partnership Game ϕ_3 is an ECB as there exists no profitable deviation from it. And also it is easy to see that this is the unique ECB of the game. Now lets find the ECB of the Regime Change game. First note that as d is the most preferred state for P and as the consent of P is needed for changing the regime from d, any ECB ϕ will assign $\phi(d) = (d, d, \emptyset)$. Otherwise P would simply deviate by refusing to take the action, which would be a profitable deviation. By the same argument, c is a stable state, i.e. $\phi(c) = (c, c, \emptyset)$. Given this, at state a, knowing that it is stable E will change the state to c, hence $\phi(a) = (a, c, E)$. It is easy to see that there is no profitable deviation from ϕ and hence it is an ECB.

Interested reader might jump to Section 8 that contains (i) an analysis of the one step deviation property that ECB satisfies in some classes of games and (ii) a discussion of issues regarding existence. Now I will directly move to the analysis of ECB in three general domains and study the relationships it engenders with the solution concepts of these domains. This will be followed by applications.

4 Extensive Form Games

It is easy to see that an extensive form game of perfect information is an extended coalitional game.

Definition 5. *Extensive Form Games of Perfect Information*
An extended coalitional game Γ is an extensive form game if the graph of Γ is a tree and for all $z \in Z$

- *Either $A_z = \{(z, z, \emptyset)\}$ (i.e. z is a terminal node) or for all $(z, z', S) \in A_z$ we have $S = \{i\}$ for the same $i \in N$ (i.e. only one individual is active).*

- *For $h', h'' \in H_z$ and $i \in N$, $h' \succeq_i h''$ iff $(h, h') \succeq_i (h, h'')$ for any path h that ends at z.*

It turns out that in finite horizon extensive form games ECB satisfies the *one step deviation property*, which immediately implies that ECB is equivalent to (pure strategy) subgame perfect equilibrium in finite horizon games.

Unlike subgame perfect equilibrium, the one step deviation property of ECB is not inherited in infinite horizon games that are continuous at infinity, such as games with discounting. In games that are continuous at infinity, under the subgame perfect equilibrium any profitable infinite deviation can be replaced with a profitable finite deviation. This is done by truncating the deviation after some period T as the payoffs get less and less important in future periods. But under an ECB this may not be the case, because whatever T we truncate the deviation at, the deviation should also increase the payoffs of the deviators at period $T-1$. So, we might not be able to replace an infinite deviation with a finite one. Nevertheless, it is easy to establish that ECB always refines subgame perfect equilibrium.

4.1 Finite Horizon Extensive Form Games

We say that ECB satisfies the one step deviation property if existence of a profitable deviation implies the existence of a profitable deviation involving actions stemming from the same node.

Definition 6. *One Step Deviation Property*

- *A deviation is a one step deviation if every action involved in the deviation stems from the same node.*

- *ECB satisfies the one step deviation property on an extended coalitional game Γ if whenever there exists a profitable deviation there also exists a profitable one step deviation.*

It turns out that finite horizon extensive form games is one domain in which ECB satisfies this property. For a more thorough analysis of the one step deviation property see Section 8.1.

Lemma 1. *ECB satisfies the one step deviation property in finite horizon extensive form games.*

The proof, as well as all other proofs, is in the Appendix. The idea behind the proof is simple. Suppose that S has a profitable deviation from a coalitional behavior ϕ. As the game is finite and acyclic we can find a node z such that the action from z has changed after the deviation, but actions at any node that is reachable from z has not changed. But then S can simply change its action at z, which would be a profitable one step deviation.

Subgame perfect equilibrium also satisfies the one step deviation property in finite horizon games, hence the equivalence between the two solution concepts follow.

Proposition 1. *Let Γ be a finite horizon extensive form game. Then ϕ is an ECB iff ϕ is a pure strategy subgame perfect equilibrium.*

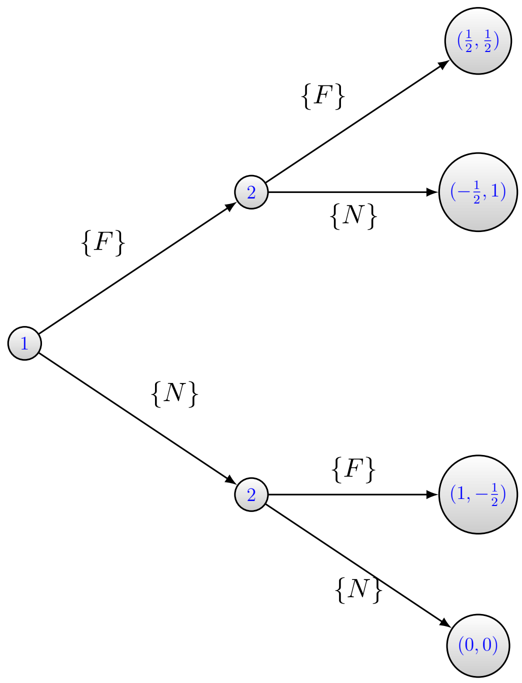


Figure 5: Favor Exchange

The result might seem counterintuitive given that a subgame perfect equilibrium outcome might Pareto dominate another and ECB allows for coalitional deviations. The following example should clear up the confusion.

Example 5. *Favor Exchange*

Player 1 *can do a favor for player* 2 *or not. Following player* 1*'s decision player* 2 *decides whether to do a favor for player* 1 *or not. A player that receives a favor gets a utility of* 1 *and a player who provides a favor incurs a cost of* $\frac{1}{2}$*. The game is represented in Figure 5.*

In this game under the unique ECB (also the unique subgame perfect equilibrium), each player chooses $\{N\}$ *at any node. There is a deviation from this ECB by* $S = \{1, 2\}$ *in which both of the players choose* $\{F\}$ *instead of* $\{N\}$*, however the deviation is not profitable as when player* 2 *is choosing between* $\{F\}$ *and* $\{N\}$ *he has no incentive choose* $\{F\}$ *over* $\{N\}$*, i.e. the deviating coalition is not better off at every node at which an action changes.*

4.2 Infinite Horizon Extensive Form Games

One step deviation property of ECB is not inherited in infinite horizon extensive form games even when the game is continuous at infinity. As a result, the equivalence breaks down. The following example of the infinitely repeated favor exchange demonstrates this.

Example 6. *Infinitely Repeated Favor Exchange*

Suppose that the players repeatedly play the favor exchange game in Example 5. At each period they get the payoffs of the favor exchange game played in that period and the payoffs are discounted with a common discount factor $\delta \in (0, 1)$*.*

There is a subgame perfect equilibrium of this game in which each player chooses $\{N\}$ *in each period. But, this subgame perfect equilibrium is not an ECB for* δ *big enough. This is because there is a profitable deviation by* $S = \{1, 2\}$ *in which the players change* *to playing $\{F\}$ at each node of the game. The deviation is profitable since at every node at which S changes its action, both of the players are better off.*

Note the difference between this deviation and the similar deviation in the finite game. In the finite game, the last player to deviate had no incentive to make the deviation. However, here there is no last player to deviate, as a result each player has an incentive to keep his part of the deviation at every node.

The example above also shows that unlike subgame perfect equilibrium, under ECB the one step deviation property no longer holds in infinite horizon extensive form games with discounting. Nevertheless, the lemma below shows that ECB is a refinement of subgame perfect equilibrium.

Lemma 2. *If ϕ is an ECB for an extensive form game then ϕ is a subgame perfect equilibrium.*

With the same logic we used to prove that ECB satisfies one step deviation property in finite horizon games, we can also show here that the existence of a finite profitable deviation implies the existence of a profitable one step deviation. Hence, if a profitable coalitional deviation that leads to an infinite path of play at some node at which an action changes does not exist then the subgame perfect equilibrium should be an ECB.

Proposition 2. *Let Γ be an extensive form game. Then ϕ is an ECB iff ϕ is a subgame perfect equilibrium that is immune to coalitional deviations to any coalitional behavior ϕ' where $\phi'(z^*) \neq \phi(z^*)$ at some $z^* \in Z$ and there exists infinitely many $z \in \sigma'(z^*)$ for which $\phi(z) \neq \phi'(z)$.*

The proposition is useful in finding the ECBs of a wide range of infinite horizon games. The following is an example.

Example 7. *Rubinstein's Bargaining Game (Rubinstein (1982))*

Two players are negotiating on how to split a pie of size 1*. In period* 0*, player* 1 *makes an offer, if player* 2 *accepts then the game ends and the pie is split. Otherwise the game moves on to period* 1 *and player* 2 *makes an offer. This goes on ad infinitum. If an agreement $(x, 1-x)$ is reached at period t then player* 1 *gets a utility of $\delta^t x$ and player* 2 *gets $\delta^t(1-x)$, where $\delta \in (0,1)$ is the common discount factor.*

As is well known, this game has a unique subgame perfect equilibrium in which player 1 *always proposes $(\frac{1}{1+\delta}, \frac{\delta}{1+\delta})$ and accepts any proposal offering him at least $\frac{\delta}{1+\delta}$. Player* 2 *always proposes $(\frac{\delta}{1+\delta}, \frac{1}{1+\delta})$ and accepts any proposal offering him at least $\frac{\delta}{1+\delta}$.*

By Proposition 2, to find the ECBs of this game it is enough to check whether the subgame perfect equilibrium is immune to coalitional deviations that lead to an infinite path of play. As any deviation leading to an infinite path of play leads to a payoff of $(0,0)$ we have that the subgame perfect equilibrium is immune to coalitional deviations and the game has a unique ECB, which is the unique subgame perfect equilibrium of the game.

The results up to know might have left the impression that the efficiency problem, which is obviously present in finite horizon games, might be solved in infinite horizon games. The following example demonstrates that this is not the case.

Example 8. *Multiperson Bargaining Game*

This is the extension of the above model to $n \geq 3$ players. $n \geq 3$ players are negotiating on how to split a pie of size 1*. In period* 0*, player* 1 *makes an offer and all other players* *sequentially accept or reject the offer. If everyone accepts the game ends, otherwise in period* 1 *player* 2 *makes an offer and all other players sequentially accept or reject. This goes on ad infinitum. If an agreement* $(x_1, x_2, ..., x_n)$ *is reached at period* t *then player* i *gets a utility of* $\delta^t x_i$*, where* $\delta \in (0, 1)$ *is the common discount factor.*

It is well known that this game has multiplicity of subgame perfect equilibria including ones that involve delay (see Osborne and Rubinstein (1990)). But, as any deviation leading to an infinite path of play leads to a payoff of $x_i = 0$ *for all* $i \in N$ *we have that each subgame perfect equilibrium is immune to coalitional deviations. Hence every subgame perfect equilibrium is also an ECB and the subgame perfect equilibria that involve delay are inefficient.*

5 Characteristic Function Games

A characteristic function game is a pair (N, V), where N is the finite set of players and for each coalition $S \subseteq N$, $V(S) \subseteq R^S$ denotes the set of payoff vectors achievable by coalition S. A coalition structure P is a partition of N. A state z is a pair (x, P) where P is a coalition structure and x is an allocation that satisfies $x_S \in V(S)$ for all $S \in P$. Let Z denote the set of all states. The core is probably the most well-known solution concept defined for characteristic function games. It is the set of states that no coalition can improve upon.

Definition 7. *The core of the game* (N, V) *is defined as*

$$C(N, V) = \{(x, P) \in Z| \text{ there does not exist } S \subseteq N \text{ and } y_S \in V(S) \text{ such that } y_S > x_S{}^9\}$$

To study ECB we need to define the extended coalitional game that corresponds to a characteristic function. I will use the convention that the nodes of the game correspond to states, where a state is an allocation and a coalition structure. The actions have already been implicitly defined in the definition of the core, in particular the definition assumes that $S \subseteq N$ can induce any allocation that is feasible for it, i.e. $((x, P), (y, P'), S) \in A_{(x,P)}$ iff $y_S \in V(S)$. But, this definition is not satisfactory. The reason is that S is able to dictate the payoffs of other coalitions and the coalition structure of $N \setminus S$, which is intuitively unreasonable.[10] This is not considered in the definition of the core, because it is a myopic concept and how the allocation is determined for the nondeviators and the coalition structure formed by the nondeviators is inconsequential (For a more complete discussion see (Ray and Vohra (2015)).

Building upon these I will require that for any $(x, P) \in Z$, the set of actions satisfy the three conditions below, which are taken from Ray and Vohra (2015).(Also see Konishi and Ray (2003) and Koczy and Lauwers (2004), who use similar conditions.)[11]

1. $((x, P), (x, P), \emptyset) \in A_{(x,P)}$
2. If $((x, P), (y, P'), S) \in A_{(x,P)}$ then $y_S \in v(S)$ and if $T \in P$ is such that $S \cap T = \emptyset$ then $T \in P'$ and $x_T = y_T$

[9] $y_S > x_S$ if $y_i > x_i$ for all $i \in S$.

[10] Which also leads to quite perverse results, see Ray and Vohra (2015) for a discussion.

[11] Note that Ray and Vohra (2015) define these conditions for an abstract game (see Section 6), whereas here they are defined on an extended coalitional game.

3. For all $(x, P) \in Z$, $T \subseteq N$ and $z_T \in V(T)$ such that either $z_T \neq x_T$ or $T \notin P$, there exists $((x, P), (y, P'), T) \in A_{(x,P)}$ such that $T \in P'$ and $y_T = z_T$.

Note that these are mild conditions that only take into consideration the drawbacks stated in the paragraph above. The first condition states that it is possible to stay in every state. The second condition states that when a coalition deviates from an outcome it has to get something feasible for itself and it cannot dictate the payoffs and structures of the coalitions that are unrelated to it. And the third condition states that if a payoff z_T is feasible for a coalition T ($z_T \in V(T)$), then T should be able to get z_T or if a coalition T has not formed then T should be able to form. This third condition is already incorporated in the definition of the core, so the only additional condition we put is the second condition, which merely restricts a coalition from determining the unrelated coalitions' payoffs and structure.

Hence, an extended coalitional game that corresponds to a characteristic function game is $\Gamma = \{N, Z, \{A_z\}_{z \in Z}, \{u_i\}_{i \in N}\}$, where N is the set of players, Z corresponds to all states and $\{A_z\}_{z \in Z}$ is any set of actions that satisfy the restrictions above. The only undefined ingredient is the utility functions on the action sequences. This might be defined in a multitude of ways and I will look at the ECBs under the following assumptions.

Definition 8. *Let $h = \{(x^k, P^k)\}_{k=0,1\ldots K} \in H$ be a generic terminal path.*[12]

- ***Myopia:*** *For all $i \in N$ and $h \in H$ we have $u_i(h) = x_i^1$.*
- ***Foresight:*** *For all $i \in N$ and $h \in H$ we have $u_i(h) = x_i^K$ if $K < \infty$, if $K = \infty$ then $u_i(h) = max\ v(\{i\})$, i.e. in case of perpetual disagreement individuals get their endowments.*
- ***Discounted Utilities:*** *There is a common discount factor $\delta \in (0, 1)$. For all $i \in N$ and $h \in H$ we have $u_i(h) = \sum_{k=0,..,K} \delta^k x_i^k + \sum_{k=K+1,...} \delta^k x_K$*

Under myopia, players only care about the immediate consequence of their actions. Under foresight, players only care about the final allocation the action sequence leads to and they dislike ongoing negotiations. Under discounted utilities, players discount the utilities of the states they visit along an action sequence.

Given a coalitional behavior, we can define a stable state as a state at which no action is taken, i.e. a state z is stable if $\phi(z) = (z, z, \emptyset)$. Let $S(\phi)$ denotes the set of all stable states.

5.1 Myopia

The core is a myopic concept; the coalitions do not consider that their deviation might be followed by further deviations. Therefore, we might expect a relationship between the stable outcomes of a myopic ECB and the core. The following lemma shows that this is indeed the case.

Lemma 3. *Suppose the players are myopic and ϕ is an ECB.*

- *If $(x, P) \in Z$ is stable under ϕ then $(x, P) \in C(N, V)$.*

[12]There is a slight abuse of notation here, that I am ignoring the initiators of actions.

- *If* $(x, P) \in C(N, V)$ *and* $(x, P) \notin S(\phi)$ *then* $\phi((x, P)) = ((x, P), (x', P'), T)$ *for some* $(x', P') \in Z$ *and* $T \subseteq N$*, where* $x'_i \geq x_i$ *for all* $i \in T$ *and* $x'_k = x_k$ *for at least one* $k \in T$*.*

The lemma states that if players are myopic then any stable state under any ECB should be a core state. Furthermore if a core state is not stable then it should be due to indifference, that some players are taking an action although they are indifferent between staying at the state and the immediate consequence of the action.

5.2 Foresight

Given the myopia of the core, the above lemma is not surprising. But it is surprising that a complete core characterization can be obtained through ECB under foresight. In particular, under foresight ECBs with a single stable outcome completely characterize the core. It should be noted that the the characterization is only for ECBs with a single stable outcome, therefore the core and ECB under foresight can still make different predictions (see Example 9 below for an example of an ECB with multiple stable outcomes that are disjoint from the core).

For any coalitional behavior ϕ and $z \in Z$, let $\mathcal{T}(\sigma(z))$ denote the terminal state of the path of play corresponding to ϕ at z if $\sigma(z)$ is finite, otherwise let $\mathcal{T}(\sigma(z)) = \emptyset$.

Proposition 3. *Under foresight,*

- *If* $(x^*, P^*) \in C(N, V)$*, then there exists an ECB* ϕ *such that* $\mathcal{T}(\sigma(x, P)) = (x^*, P^*)$ *for every* $(x, P) \in Z$*.*

- *If* ϕ *is an ECB such that* $\mathcal{T}(\sigma(x, P)) = (x^*, P^*)$ *for every* $(x, P) \in Z$ *then* $(x^*, P^*) \in C(N, V)$*.*

The proposition states that any core state can be supported as the single stable outcome of an ECB under foresight and if a state can be supported as the single stable outcome of an ECB under foresight then it must be a core state. Hence, ECBs under foresight with a single stable outcome completely characterize the core.

The value of this result can be seen in the following three points. First, a minimal set of assumptions lead to a strong result. In particular, notice that no condition is imposed on the characteristic function game, neither superadditivity nor any other widely used assumption is imposed. Furthermore the result is valid for both transferable utility and nontransferable utility games. The only assumptions made are the three assumptions on the actions, which are both weak and intuitive. Even those assumptions don't specify how coalitions behave following a deviation. For example, when S deviates, we don't make any assumption on how any coalition T, where $S \cap T \neq \emptyset$ behaves, T may dissolve or stay together or form any other coalition structure within itself; all of these would satisfy our assumptions on the actions.

Second, this result states that a farsighted solution concept completely characterizes a well-known myopic concept, the core. An allocation is in the core if there is no myopic objection to it, but an allocation can be supported as the single stable outcome of an ECB under foresight, if we can find a path from each outcome to the allocation such that no coalition would be better off by deviating from these set of paths. The two arguments are quiet different from each other, but they lead to the same predictions.

Finally, this result complements other results in the literature that are close in spirit to this result. Other results that show that the core incorporates foresight include Ray (1989), Diamantoudi and Xue (2003), Konishi and Ray (2003), Mauleon, Vannetelbosch and Vergote (2011) and Ray and Vohra (2015).[13] Ray (1989) shows that core is immune to nested objections. The results in Diamantoudi and Xue (2003), Mauleon, Vannetelbosch and Vergote (2011) and Ray and Vohra (2015) concern the farsighted stable set (see Section 6.1.2). Whereas Konishi and Ray (2003) study a real time dynamic process when the discount factor goes to 1.

Unlike any of these results, Proposition 3 is a complete characterization of the core in any characteristic function game. The reasons why the above results do not provide a complete characterization differ. Most of the above papers are about the farsighted stable set (see Section 6.1.2). The result above shows that the only reason they do not completely characterize the core is because external stability in the farsighted stable set is defined with strict preference instead of weak preference. That is, as shown in the proof of the theorem (see Appendix), in any characteristic function game there exists a path from any state to any core state such that every coalition moving along the path weakly prefers the final state to the state it replaces. However, this weak preference cannot be replaced with strict preference and therefore the core state might not satisfy external stability with respect to the indirect dominance relation.

The concept Konishi and Ray (2003) uses is more related to the next section as they consider a dynamic process in which the utilities are discounted. There we will see that their core characterization partly relies on not allowing for some reasonable deviations.

Proposition 3 implies that if a characteristic function game has an empty core, then we cannot find an ECB for that game with a single stable outcome under foresight. But this does not mean that an ECB does not exist in such a game. Below is an example of an ECB under foresight defined for a characteristic function game with an empty core.

Example 9. *There are 3 players. $V(S) = 0$ if $|S| = 1$ and $V(S) = \{x| \sum_{i \in S} x_i \leq 1\}$ otherwise. The core of this game is empty. Now, I will describe an ECB ϕ for this game. Let $x_1 = (0, \frac{1}{2}, \frac{1}{2})$, $x_2 = (\frac{1}{2}, 0, \frac{1}{2})$ and $x_3 = (\frac{1}{2}, \frac{1}{2}, 0)$. Let $P_1 = \{\{1\}, \{2, 3\}\}$, $P_2 = \{\{1, 3\}, \{2\}\}$ and $P_3 = \{\{1, 2\}, \{3\}\}$. Finally define $z_i = (x_i, P_i)$ for $i = 1, 2, 3$. If z is a state where player i gets more than $\frac{1}{2}$, then ϕ contains the action from z to z_i. If z is a state where nobody gets more than $\frac{1}{2}$ and $z \neq z_i$ for $i = 1, 2, 3$, then ϕ contains the action from z to z_1. z_1, z_2 and z_3 are stable. It is easy to see that ϕ is an ECB under foresight.*

Note the logic underlying the ECB in this example: z_1 is not in the core, because $S = \{1, 2\}$ has a profitable deviation, say to $(\frac{1}{4}, \frac{3}{4}, 0)$. But, this is a myopic deviation, if everybody is expected to act as the above ECB specifies then player 2 should be afraid to make this deviation knowing that he would be punished by an outcome (z_2) which gives him 0, while increasing the payoff of his co-deviator (player 1) to $\frac{1}{2}$.

5.3 Discounted Utilities

Analyzing the ECB under the assumptions of myopia and foresight is particulary simple, as under these assumptions players only care about a certain state on each path of play.

[13] Green(1974), Feldman(1974), Koczy and Lauwers (2004) and Sengupta and Sengupta(1996) show how myopic objections lead to the core in specific environments. Although these papers are also related, they are fundamentally different from the current paper, since players are not assumed to be farsighted.

When there is a discount factor, this is no longer the case. Nevertheless, it is easy to see that Lemma 3 is still valid if $\delta \to 0$, but the same cannot be said about Proposition 3.

Example 10. *There are 3 players.* $V(S) = 0$ *if* $|S| = 1$, $V(\{1,2,3\}) = (2,2,2)$, $V(\{1,2\}) = (10,1)$, $V(\{2,3\}) = (10,1)$ *and* $V(\{1,3\}) = (1,10)$. *In this game the core is unique and is composed of* (x, N), *where* $x = (2,2,2)$. *But, it cannot be supported as the single stable outcome of an ECB when* $\delta \to 1$. *The reason is that when players cycle between the coalition structures* $(12,3)$, $(23,1)$ *and* $(13,2)$ *everybody receives a higher payoff than the one they get under the core allocation.*

This example also demonstrates a feature of Konishi and Ray (2003)'s result. In the above example, under their solution concept (see Section 6.2 for the definition) the core allocation can be supported as the unique stable outcome when $\delta \to 1$. This is because, under their solution concept players cannot coordinate to impose the deviation that results in the preferred cycle. Hence their core characterization partially relies on ruling out some intuitive deviations.

Finally, the other part of Proposition 3 still holds when $\delta \to 1$.

Proposition 4. *Suppose players use a common discount factor* $\delta \in (0,1)$ *to evaluate the utility of a path. If* $\delta \to 1$ *and if* ϕ *is an ECB such that* $\mathcal{T}(\sigma(x,P)) = (x^*, P^*)$ *for every* $(x,P) \in Z$ *then* (x^*, P^*) *is in the core.*

6 The Abstract Game

Most of the static and the dynamic solution concepts discussed in the Literature Review are defined on the abstract game. ECB can be defined on the abstract game both as a static concept and a dynamic concept. In this section we will do this and compare the resulting ECB to other solution concepts.

An abstract game is defined as $\Gamma = \{N, Z, \{v_i\}_{i\in N}, \{\stackrel{S}{\rightarrow}\}_{S\subseteq N, S\neq\emptyset}\}$ (see, for example, Chwe (1994), Rosenthal (1972) and Xue (1998)). Where N is the set of players, Z is the set of states, $\{v_i\}$ is player i's utility function defined on the set of states and $\{\stackrel{S}{\rightarrow}\}_{S\subseteq N, S\neq\emptyset}$ are effectiveness relations defined on Z. The effectiveness relation $\{\stackrel{S}{\rightarrow}\}$ describes what coalition S can do at every state, i.e. $a \stackrel{S}{\rightarrow} b$ for $a, b \in Z$ iff when a is the status quo coalition S can change state a with state b.

The definition of an abstract game and an *extended coalitional game* are similar, the major difference being that in an extended coalitional game utilities are defined over the paths, whereas under an abstract game utilities are defined on the nodes (states).

A simple way to see the impact of this difference is to look at an infinite horizon extensive form game. We have already seen that this is an extended coalitional game, but it is not an abstract game. The reason is that the actions (or effectiveness in the language of the abstract game) are defined over non-terminal nodes, hence the set Z is the set of nonterminal nodes and under an abstract game utilities are defined over Z. But in an infinite horizon extensive form game there are no utilities defined on the intermediate nodes, they are defined over paths just like in an extended coalitional game.

For finite extensive form games, Xue (1998) uses the trick of assigning a very low utility to nonterminal nodes, thereby forcing the players to move to terminal nodes. But even such a trick is not applicable in an infinite horizon game. Hence an infinite horizon extensive form game is not an abstract game.[14]

All of the ingredients of an abstract game admit an obvious translation to the extended coalitional game except for the utilities. In particular the set of players and the set of states is the same and the set of actions correspond to the effectiveness relation. Hence, let $\{N, Z, \{A_z\}_{z\in Z}, \{u_i\}_{i\in N}\}$ be the extended coalitional game corresponding to an abstract game, where $(z, z, \emptyset) \in A_z$ for all $z \in Z$ and $(z, x, S) \in A_z$ iff $z \xrightarrow{S} x$ for $S \neq \emptyset$.

For the utilities, the static approach assumes that players only care about the final states their actions lead to, which corresponds to foresight defined in Section 5. Whereas the dynamic approach assumes that players care about the discounted payoff of all the states they visit along a path, which corresponds to discounted utilities on the paths.For convenience, the definitions are replicated below.

Definition 9. *Let $h = \{(z_k, z_{k+1}, S_k)\}_{k=0,1\ldots K} \in H$ be a generic terminal path.*

- ***Foresight:*** *For all $i \in N$ and $h \in H$ we have $u_i(h) = v_i(z_K)$ if $K < \infty$, if $K = \infty$ then $u_i(h) = -\infty$, i.e. players dislike ongoing negotiations.*
- ***Discounted Utilities:*** *There is a common discount factor $\delta \in (0, 1)$. For all $i \in N$ and $h \in H$ we have $u_i(h) = \sum_{k=0,..,K} \delta^k v_i(z_k) + \sum_{k=K+1,\ldots} \delta^k v_i(z_K)$*

I will call the ECB corresponding to the assumption of foresight, the farsighted ECB (FECB) and the one corresponding to discounted utilities, the dynamic ECB (DECB). Both approaches will be considered in order. The analysis focuses on the relationship of the concepts to the ECB. For a treatment that shows the various strengths and weaknesses of the concepts see Herings, Mauleon and Vannetelbosch (2004), Dutta and Vohra (2015), Ray and Vohra (2014) and Xue (1998).

6.1 The Static Approach

6.1.1 Largest Consistent Set (LCS)

LCS, developed by Chwe (1994) is a conservative solution concept, it tries to rule out with confidence. Its definition is based on the indirect dominance relation on the set of states.

Definition 10. *Indirect Dominance*

$x \in Z$ indirectly dominates $y \in Z$ ($x \gg y$) if there exists $x_0, x_1, \ldots x_n \in Z$ and $S_0, S_1, S_2, \ldots, S_{n-1}$ such that $x_0 = y$, $x_n = x$, $(x_i, x_{i+1}, S_i) \in A_{x_i}$ and $v_j(x_n) > v_j(x_i)$ for all $j \in S_i$, for all $i = 0, ..n-1$.

Definition 11. *Largest Consistent Set*

A set $V \subset Z$ is consistent if $a \in V$ iff for all d, S such that $(a, d, S) \in A_a$, there exists $e \in V$, where $d = e$ or $e \gg d$, such that $v_i(a) \geq v_i(e)$ for some $i \in S$. The consistent set that contains all other consistent sets is called the largest consistent set.

[14] One can take the states as strategy profiles and then study the game as an abstract game. But this would be done at the expense of information about the game.

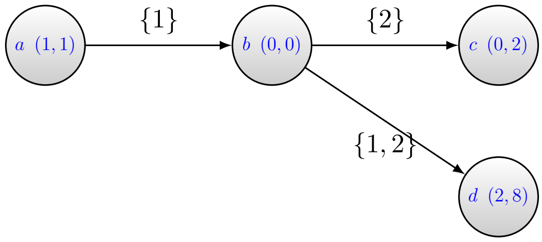


Figure 6: LCS differs from FECB (Xue (1998))

Sometimes LCS is unable to make obvious predictions as shown in the example in Figure 6 taken from Xue (1998). In this example the LCS is $\{a, c, d\}$. According to LCS, a is stable because under the assumptions of the LCS players are pessimistic and player 1 is afraid that player 2 will move to state c following state b. We see that instead of considering the best course of action player 2 has at state b, player 1 forms an unreasonable expectation based on a behavioral attitude (pessimism) over player 2's actions. It is easy to see that FECB makes the 'correct' prediction in this game.[15]

Given that LCS is a conservative solution concept, one might expect the stable states under the LCS to contain the stable states of an ECB. The following proposition states that if there is no indifference then this is indeed the case.

Definition 12. *No Indifference (NI)*

An abstract game Γ satisfies NI if for all $i \in N$ and $z, z' \in Z$, where $z \neq z'$, we have $v_i(z) \neq v_i(z')$.

Proposition 5. *Let Γ be an abstract game that satisfies NI. If ϕ is an FECB of Γ then $S(\phi) \subseteq LCS$.*

The solution concepts in the static approach are defined using the indirect dominance relation which uses strict preference. Whereas under the FECB coalitions might take some actions even if they are indifferent, so we will always see a difference between these concepts and the FECB due to the different ways with which they treat indifference. I do not view this difference as essential. Therefore, not to bump on it again and again, from now on I will assume that the environment satisfies NI.

6.1.2 Farsighted Stable Set (FSS)

Farsighted stable set (see Harsanyi (1974) and Chwe (1994)) is basically von Neumann and Morgenstern's stable set defined with the indirect dominance relation.

Definition 13. *Farsighted Stable Set (FSS)*

A set $V \subseteq Z$ is an FSS if

- *For any $x, y \in V$ we have $x \not\gg y$*
- *For any $y \notin V$, there exists $x \in V$ such that $x \gg y$*

There is no relation between the FECB and the FSS. The example in Figure 7 shows that some stable outcomes of the unique FSS might be unstable in the unique FECB and some stable outcomes of the unique FECB might be unstable in the unique FSS.

[15] The unique ECB ϕ specifies $\phi(a) = (a, b, \{1\})$ and $\phi(b) = (b, d, \{1, 2\})$.

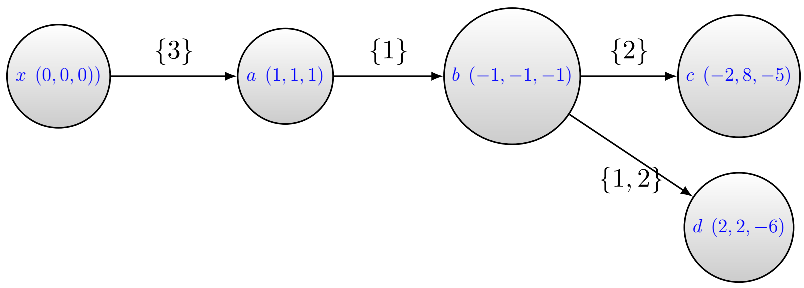


Figure 7: FSS differs from FECB

The reasonable prediction and the unique FECB ϕ of this game specifies $\phi(b) = (b, c, 2)$, $\phi(a) = (a, a, \emptyset)$ and $\phi(x) = (x, a, 3)$. However under the FSS a is not stable, because 1 optimistically and unreasonably believes that 2 will move to d with him, but this in turn makes x stable under the unique FSS. Hence, $FSS = \{x, c, d\}$. The problem is that under FSS players rely on unreasonable expectations of other players' behavior instead of forming consistent expectations and reacting optimally to it. It is easy to see that FECB avoids this drawback of the FSS.

Dutta and Vohra (2015) tries to solve the problems associated with the FSS by incorporating the solution concept with consistent expectations and the idea of maximality. Surprisingly the exercise results in a solution concept that is very similar to the FECB. Now we will look into this.

6.1.3 REFS and SREFS

REFS and SREFS (see Dutta and Vohra (2015)) blend consistent expectations with farsighted stability and one-step deviations to embed the farsighted stable set with the idea of maximality. The difference between REFS and SREFS is that in the latter a deviation is defined as a stronger concept. As the more related concept is SREFS, in this section I will restrict attention to SREFS. Using coalitional behavior rather than expectations we can restate their stability concept in the language of our framework.

Definition 14. *SREFS (Dutta and Vohra (2015))*

A set $V \subseteq Z$ is an SREFS if there exists an acyclic coalitional behavior ϕ such that $S(\phi) = V$ and

(IS) *If $x \in V$ then there does not exist y, S such that $(x, y, S) \in A_x$ and $v_i(\mathcal{T}(\sigma(y))) > v_i(x)$ for all $i \in S$.*

(ES) *If $x \notin V$ then $\sigma(x)$ is an indirect dominance path.*

(M) *If $x \notin V$ and if T is the initiator at x then there does not exist $y \in Z$ and $F \subseteq N$ with $T \cap F \neq \emptyset$ and $(x, y, F) \in A_x$ such that $v_i(\mathcal{T}(\sigma(y))) > v_i(\mathcal{T}(\sigma(x)))$ for all $i \in F$.*

The first and second conditions are interpreted as internal and external stability conditions with respect to the expectation. Whereas the third condition requires optimality of the move at any node x, where optimality is conditioned on a one-step deviation.

It turns out that under the weak assumption that actions are monotonic, in the sense that whenever a coalition S is able to take a certain action then any coalition T containing S can also take this action, FECB is equivalent to SREFS.

Definition 15. *Monotonicity of Actions (MOA)*

We say that an extended coalitional game satisfies MOA if whenever $(z, z', S) \in A_z$ for some $z \in Z$ we also have that $(z, z', T) \in A_z$ for all $T \supseteq S$.

Proposition 6. *Let Γ be an abstract game that satisfies NI and MOA then*

- *If V is an SREFS and ϕ is the coalitional behavior that supports it then ϕ is an FECB.*
- *If ϕ is an FECB then $S(\phi)$ is an SREFS supported by ϕ.*

This shows that the conditions of internal and external stability with the one-step deviation condition rules out every possible profitable deviation, even those that involve multiple actions. One of the reasons why this is the case is that FECB satisfies one step deviation property (see Section 8.1). Therefore, a static solution concept defined this way doesn't really have to consider complicated deviations.

6.2 The Dynamic Approach

EPCF (See Konishi and Ray (2003) and Ray and Vohra (2014)) is the main solution concept in the dynamic approach.[16] In this section I will compare the DECB to the EPCF.

6.2.1 EPCF

EPCF also uses consistent expectations just like SREFS and ECB. The difference of EPCF and SREFS (apart from one being a static and the other being a dynamic concept) is that the former is directly defined as an expectation (a coalitional behavior) that is immune certain deviations. Using coalitional behavior, we can restate the definition of EPCF under NI in the language of our framework.[17]

Definition 16. *EPCF*

A (deterministic) EPCF[18] *is a coalitional behavior ϕ such that for all $x \in Z$*

- *if $\phi(x) = (x, y, S)$, where $y \neq x$ then $\sigma(y) \succ_S \sigma(x)$ and there does not exist z with $(x, z, S) \in A_x$ and $\sigma(z) \succ_S \sigma(y)$.*
- *if x is such that there exists y, S with $(x, y, S) \in A_x$ and $\sigma(y) \succ_S \sigma(x)$ then $\phi(x) \neq (x, x, \emptyset)$*

where for any $z, z' \in Z$, $\sigma(z) \succ_i \sigma'(z)$ iff the discounted utility of the former is greater.

[16] Although similar and they have the same name, the concepts in these two papers are different. In particular the EPCF of Ray and Vohra (2014) requires more information, such as a protocol, than the EPCF of Konishi and Ray (2003). That is why I will compare the DECB to the EPCF of Konishi and Ray (2003) which is defined on the abstract game.

[17] The assumption of NI is needed here for an altogether different reason. In particular in the original definition of EPCF, Konishi and Ray allow a coalition to move at state x even if it is indifferent between moving or staying at x. But they do not allow this if there exists a coalition that can move at x and that would strictly improve by taking an action at x. Whereas under a DECB this is also allowed.

[18] Konishi and Ray (2003)'s EPCF can also be stochastic, as mixing is not included in the definition of an ECB I restrict attention to deterministic EPCFs.

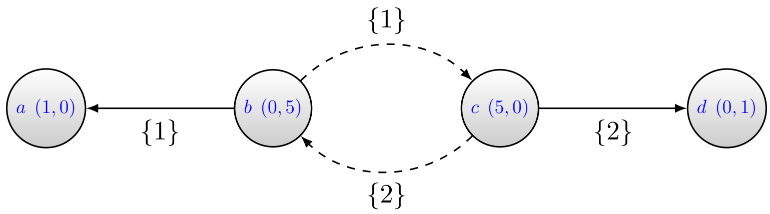


Figure 8: Profitable Deviation to a Cycle

The definition is similar to the definition of DECB with the major difference being that this definition does not consider deviations that involve multiple actions.[19] Unlike FECB in the static approach, DECB does not satisfy the one step deviation property. This means that the restriction to one-step deviations in the above definition is with loss of generality. In particular, there might exist a deviation to a cycle that might make the deviators better off.

An example is given in Figure 8, here at node b, player 1 moves to a and settles for a payoff of 1. Similarly player 2 moves from c to d and settles for a payoff of 1. There is no profitable one-step deviation from this coalitional behavior for δ big enough. But, there is a profitable deviation by the coalition $\{1, 2\}$ to the cycle. Hence for δ big enough the depicted coalitional behavior is not a DECB, but it is an EPCF.

Finally, it is easy to establish that every DECB is an EPCF. The following proposition summarizes the results.

Proposition 7. *If ϕ is a DECB then it is an EPCF, but an EPCF may not be a DECB.*

6.3 The Static, the Dynamic and the Noncooperative Approach

Propositions 2, 6 and 7 bring the noncooperative approach and the static and the dynamic approaches together. In particular they show that ECB can be used to unify all of these three approaches; as it can be defined as a solution concept in each of these approaches and also because it is closely related to a solution concept in each of these three approaches. Hence, these propositions substantiate the claim made in the Literature Review that ECB stands where the noncooperative, the static and the dynamic approaches meet.

7 Applications

Up to now, we looked at general classes of games and the relations ECB engenders with the popular solution concepts in these classes. In this section, we will look at how ECB can be used in applications. To this end, I will define a class of games, *beeline games*, and I will characterize the ECBs of beeline games and finite acyclic games.

These two classes of games can be used to analyze a wide range of situations including network formation (Aumann and Myerson (1988)), sequential formation of binding agreements (Bloch (1996), Ray and Vohra (1999)), dynamic club formation (Barbera, Maschler and Shalev (2001), Roberts (2015)) and various political games (Acemoglu, Egorov and Sonin (2008,2012)). Most of these cited works have used subgame perfect equilibrium and its refinements to study the problem at hand. Therefore, the authors

[19]Another difference is that the definition of the deviations are in general weaker under EPCF. Furthermore, Konishi and Ray (2003) restrict attention to games with a finite number of states.

needed to complement the description of the game with the rules of coalition formation and in most cases they needed to refine the concept of subgame perfect equilibrium to get reasonable predictions.

This inevitably brings a level of arbitrariness. The analysis in this section shows that this is unnecessary, that there is a simpler and consistent way to approach these problems without specifying an arbitrary negotiation procedure and without using a wide range of different solution concepts. In particular, ECB can be directly applied to these problems and it does not need the description of the negotiation procedure to make a prediction.

7.1 Beeline Games

The analysis of beeline games has been especially influenced by Acemoglu, Egorov and Sonin (2008, 2012). In the latter of these, the authors provide a general framework to study dynamic collective decisions, a situation ECB is developed for. With the use of ECB we can repeat the exercise they do in a more general class of games, namely *beeline games.*

Beeline games are finite games which take the convention that the nodes correspond to states. The defining feature of beeline games is that there always exists a shortcut between two nodes that are reachable from each other. More precisely, whenever node y is reachable from node x through a path initiated by $S \subseteq N$, then y is also reachable from x through a single action whose initiator is S.

Definition 17. *Beeline Games*

Let Γ be an extended coalitional game. Γ is a beeline game if Z is finite, $(z, z, \emptyset) \in A_z$ for all $z \in Z$ and for all $x, y \in Z$ whenever y is reachable from x through a path initiated by $S \subseteq N$ we also have that $(x, y, S) \in A_x$.[20]

The definition of the game leaves the preferences out. In the particular beeline game Acemoglu, Egorov and Sonin (2012) consider, they assume that players have utilities defined over the set of states (nodes) and the utility of a path is the discounted utility of the states visited along the path, where the common discount factor δ is arbitrarily close to 1, i.e. $\delta \to 1$. Furthermore, they assume a cost to taking actions (changing states) such that whenever an action is taken players receive a utility lower than the utility of any state.

I will use a simpler, but equivalent way to define the preferences. In particular, the utility of each finite terminal path will be associated with a single utility corresponding to the final node of the terminal path minus the length of the path times ϵ, where ϵ is an arbitrarily small positive number. And each infinite length terminal path will have a utility of $-\infty$. It is easy to see that this specification will lead to the same predictions as the above specification.[21]

Let v_i denote the utility of $i \in N$ over the set of nodes Z and let u_i denote the utility over terminal paths. Then we have that $u_i(h) = lim_{\epsilon \to 0}\{v_i(\mathcal{T}(h)) - \epsilon L(h)\}$ if h is of finite length, otherwise $u_i(h) = -\infty$. Where $L(h)$ denotes the length of path h and $\mathcal{T}(h)$ denotes the final node of path h.

[20] y is reachable from x through a path initiated by S if there exists $z_1, z_2, ..., z_n$ and $S_1, S_2, ..., S_{n-1}$ such that $z_1 = x$, $z_n = y$, $S_1 = S$ and $(z_i, z_{i+1}, S_i) \in A_{z_i}$ for all $i = 1, ..., n-1$

[21] In both of the specifications, infinite paths provide the lowest utility and finite paths can be compared based on the terminal node if they have terminal nodes that provide different utilities, otherwise shorter paths are always preferred.

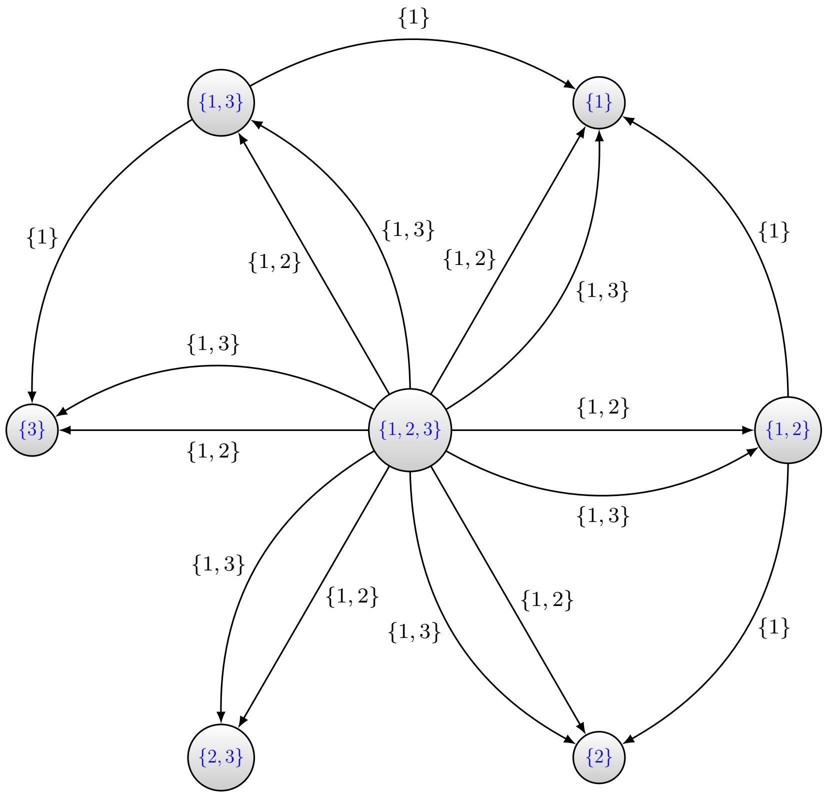


Figure 9: Example 11

There are many situations encountered in the literature that can be represented as a beeline game, the examples below demonstrate this.

Example 11. ***(Acemoglu, Egorov and Sonin (2008))*** *There is a society composed of $N = \{1, 2, .., n\}$, where each individual $i \in N$ is endowed with a political power $\gamma_i \geq 0$. Coalition $S \subseteq T$ is said to be winning within coalition T if $\sum_{i \in S} \gamma_i > \alpha \sum_{i \in T} \gamma_i$, where $\alpha \geq \frac{1}{2}$. For any coalition $T \subseteq N$, let $\mathcal{W}_T$ denote the set of winning coalitions within T.*

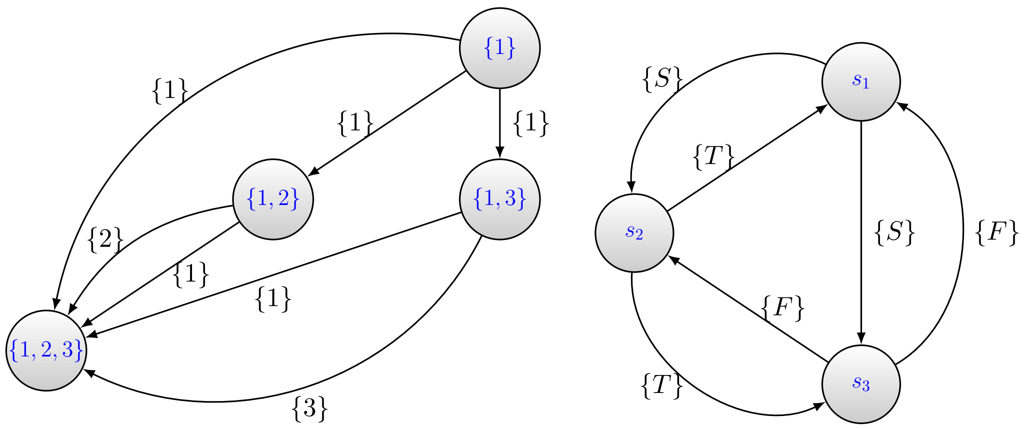
At the start of the game the society is intact. Any coalition S that is winning within N can choose to eliminate any $T \subseteq N$. But once T is eliminated, any coalition that is winning within $N \backslash T$ can choose to eliminate any set of remaining players. This continues until a self-enforcing coalition forms. v_i is defined over all subsets of N for all $i \in N$.[22]

The game can be readily formalized as an extended coalitional game, where the nodes correspond to the set of remaining players and the actions are represented with the arcs. This is done in Figure 9 for the case where $N = \{1, 2, 3\}$, $\gamma_1 = 2$, $\gamma_2 = \gamma_3 = 1$ and $\alpha = \frac{1}{2}$.

Example 12. ***(Barbera, Maschler and Shalev (2001))*** *There is a society $N = \{1, 2, .., n\}$ and a club $F \subseteq N$. Everybody in the society wants to be a part of the club. Any member of the club $i \in F$ can admit new members $S \subseteq N \setminus F$ to the club. Once the new members are admitted, any member of the expanded club can admit any set of individuals that are not a part of the club. This continues until a stable club forms. v_i is defined over supersets of F for all $i \in N$.*[23]

[22] Acemoglu, Egorov and Sonin (2008) make the assumption that an individual cannot eliminate himself: 'for simplicity, we assume that an individual cannot propose to eliminate himself' (Acemoglu, Egorov and Sonin (2008), p.8). I do not make this assumption.

[23] In their model there is a fixed number of periods in which admission decisions can be made. I assume



(a) Club Formation

(b) Example 13

The game can be represented similar to Example 11*. The representation of the game when* $N = \{1, 2, 3\}$ *and* $F = \{1\}$ *is available in Figure 10a.*

Example 13. ***(Acemoglu, Egorov and Sonin (2012))*** *There is a finite set of states* Z*. Associated with each state* s *is a set of winning coalitions* $\mathcal{W}_s$*. The game starts at an exogenously given state* s_0*. At* s_0 *any coalition* $S \in \mathcal{W}_{s_0}$ *can choose a new state* $s' \in Z \setminus s_0$ *to transition to. Following this any* $S \in \mathcal{W}_{s'}$ *can choose another state to transition to. This continues until a stable state is reached.* v_i *is defined over* Z *for all* $i \in N$*.*

This game can be represented with a complete directed graph, where the nodes correspond to the states and the arcs are labeled with the winning coalitions that can change the state. This is done in Figure 10b for the case when $Z = \{s_1, s_2, s_3\}$*,* $\mathcal{W}_{s_1} = S$*,* $\mathcal{W}_{s_2} = T$ *and* $\mathcal{W}_{s_3} = F$*.*[24]

Note that none of the examples above is a special case of another. But they are all *beeline games*. I will start with preliminary results. For $x, y \in Z$, $x \succ^D y$ (x dominates y) if there exists $S \subseteq N$ with $(y, x, S) \in A_y$ and $v(x) >_S v(y)$.[25] We say that a beeline game satisfies acyclicity of domination (AOD) if the relation $\succ^D$ does not contain any cycles.[26]

AOD is satisfied in games with veto players, for example the games in which the transition rule is dictatorship or unanimity. Furthermore it is also satisfied in a general class of games, namely all acyclic games.[27]

The following lemma states that in any beeline game, the prediction will be reached in one step and in games that satisfy AOD, the stable outcomes of any ECB will be the same.

Lemma 4. *Let* ϕ *be an ECB of a beeline game* Γ*.*

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there is no such restriction. If we put that restriction then the game is not a beeline game, but it will be a finite acyclic game, see Section 7.2.

[24]The example due to Roberts (2015), discussed in the Introduction, is a special case of this.

[25]We have $v(x) >_S v(y)$ if $v_i(x) > v_i(y)$ for all $i \in S$.

[26]I.e. for any $x_1, x_2, ..., x_n \in Z$, whenever $x_1 \succ^D x_2 \succ^D .... \succ^D x_n$ we have $x_1 \neq x_n$

[27]This is because if the domination relation has a cycle then each state within the cycle is reachable from each other. But then the game is not acyclic.

- *In ϕ the prediction is always reached in at most one step, i.e. $L(\sigma(z)) \leq 1$ for all $z \in Z$.*

- *If ϕ' is another ECB for Γ, then $\phi(z) = (z, z, \emptyset)$ iff $\phi'(z) = (z, z, \emptyset)$.*

We are ready to provide an algorithm that finds all of the ECBs of a beeline game that satisfies AOD. Suppose that the game satisfies AOD. Let $Z_0 = \text{argmax}_{\succ^D} Z$, $Z_1 = \text{argmax}_{\succ^D}\{Z \setminus Z_0\}$ and so on. Note that as $\succ^D$ is acyclic a maximal element exists as long as the set is nonempty. Continue until Z_K, where $Z_{K+1} = \emptyset$. As Z is finite, Z_K is well defined.

Proposition 8. *ϕ is an ECB of a beeline game Γ satisfying AOD iff ϕ can be obtained with the following procedure.*

1st Step: *Defining the stable outcomes $\mathcal{S}$.*

Let $\mathcal{S}_0 = Z_0$. Suppose $\mathcal{S}_n$ is defined for all $n < k$. Then for $x \in Z_k$ define

$$\mu(x) = \{z \in \{\cup_{l<k}\mathcal{S}_l\} | z \succ^D x\}$$

Let $\mathcal{S}_k = \{x \in Z_k | \mu(x) = \emptyset\}$. Continue until $\mathcal{S}_K$ is defined. Let $\mathcal{S} = \cup_{i=0,1,..,K}\mathcal{S}_i$.

2nd Step: *Defining ϕ.*

Take any $x \in Z$. Let $\phi(x) = (x, x, \emptyset)$ iff $x \in \mathcal{S}$. Otherwise let $\phi(x) = (x, z, S)$ for some z, S where $z \in \mathcal{S}$, $v(z) >_S v(x)$ and there does not exist $z' \in \mathcal{S}$, $j \in S$, $T \subseteq N$ with $(x, z', T) \in A_x$, $v(z') >_{T\cup\{j\}} v(z)$.

The algorithm is a simple recursive algorithm that pins down all of the ECBs of a beeline game satisfying AOD. It starts with the nodes that are undominated, which need to be stable as there is no state that is reachable from them that would make the initiators better off. From there it moves on to the nodes that are only dominated by an undominated node. Since these nodes are dominated by a stable node, they are unstable. This continues until the set of stable nodes is defined. Once the set of stable nodes is defined then we can assign a prediction to every nonstable state, by making sure that there exists no deviation from this prediction.

Note that existence is not guaranteed, see Section 8.2 for examples of beeline games with no ECB. Finally, I will solve Examples 13 and 11 to demonstrate the algorithm.

Dynamics and Stability of Constitutions, Coalitions and Clubs (Acemoglu et al. (2012))

Here, I will solve Example 13 defined above. Acemoglu, Egorov and Sonin (2012) keep the following assumption throughout their paper.

Assumption 1. *For any $x, y, z \in Z$ we have that*

1. *If $S, T \in \mathcal{W}_x$ then $S \cap T \neq \emptyset$.*

2. *If $S, T \in \mathcal{W}_x$, $v(y) >_S v(x)$ and $v(z) >_T v(y)$ then $z >^D x$.*

3. *The game satisfies AOD.*

The first condition rules out the possibility that two disjoint coalitions are winning in the same state. The second condition imposes that if y dominates x and if a winning coalition in x prefers z to y then z also dominates x. Finally, the last condition is the usual acyclicity condition. Under these conditions, Proposition 8 can be restated as follows.

Corollary 1. *ϕ is an ECB of Example* 13 *under Assumption* 1 *iff ϕ can be obtained with the following inductive procedure. For any $z \in Z_0$, let $\phi(z) = (z, z, \emptyset)$. Suppose ϕ is defined for all $z \in Z_n$ for all $n < k$. Then for $x \in Z_k$ define*

$$\mu'(x) = \{z \in \{\cup_{l<k} Z_l\} | \phi(z) = (z, z, \emptyset) \text{ and } z \succ^D x\}$$

Let $\phi(x) = (x, x, \emptyset)$ if $\mu'(x) = \emptyset$. Otherwise let $\phi(x) = (x, z, S)$ for some z, S where $z \in \mu'(x)$, $v(z) >_S v(x)$ and there does not exist $T \in \mathcal{W}_x$, $y \in \mu'(x)$ with $v(y) >_T v(z)$.

To see that Proposition 8 reduces down to Corollary 1, first notice that the set of stable states is obtained exactly the same way in both. So, we only need to check the $2nd$ Step of Proposition 8. First suppose that $\phi(x) = (x, z, S)$ satisfies the conditions in Proposition 8, but violates the condition in Corollary 1. Then there exists $T \in \mathcal{W}_x$, $y \in \mu'(x)$ with $v(y) >_T v(z)$. Note that by Assumption 1, $T \cap S \neq \emptyset$. But then the condition in Proposition 8 is violated, because there exists $j \in S$ ($j \in S \cap T$), $T \subseteq N$ and $y \in \mathcal{S}$ with $(x, y, T) \in A_x$, $v(y) >_{T \cup \{j\}} v(z)$.

For the other way assume $\phi(x) = (x, z, S)$ for some z, S where $z \in \mu'(x)$, $v(z) >_S v(x)$ and there does not exist $T \in \mathcal{W}_x$, $y \in \mu'(x)$ with $v(y) >_T v(z)$. Towards a contradiction assume that (x, z, S) does not satisfy the condition in Proposition 8. Then there exists z' stable, $j \in S$ with $(x, z', T) \in A_x$, $v(z') >_{T \cup \{j\}} v(z)$. Since $(x, z', T) \in A_x$, we have $T \in \mathcal{W}_x$. By the second condition in Assumption 1, we also have that $z' \in \mu'(x)$, a contradiction.

The characterization is the same as the one obtained by Acemoglu, Egorov and Sonin (2012). What is more is that we can also use Proposition 8 to come up with the characterization they obtained for Example 11, which is independent of Example 13. In particular, Example 13 allows for any transition, whereas in Example 11 only nested deviations by coalitions are permitted.

Coalition Formation in Nondemocracies (Acemoglu et al. (2008))

Here, I will solve Example 11 defined above. Acemoglu, Egorov and Sonin (2008) assume that $v_i(S)$ depends positively on i's relative strength in S. To simplify the problem I will use the following particular function, which satisfies this assumption. Note that the result would still go through if we assume the more general form that they do.[28] Let $\gamma_\emptyset = 0$ and $v_i(S) = \frac{\gamma_{S \cap \{i\}}}{\gamma_S}$ for all $i \in N$ and $S \subseteq N$ with $S \neq \emptyset$.

Let $\mathcal{C}$ denote the set of all coalitions in N and let $\mathcal{C}_X$ denote the set of coalitions that are a subset of X for some $X \subseteq N$. For any natural number k let $\mathcal{C}^k = \{X \in \mathcal{C} || X| = k\}$. Since the game is acyclic, it satisfies AOD, so we can use Proposition 8 to characterize the ECBs of the game.

The following corollary follows from Proposition 8. The proof, which is in the Appendix, is similar to the argument used above to show Corollary 1.

[28] Actually nothing in the argument changes if we look at the more general form, nevertheless I chose to stay with this specific functional form as it is intuitive and easy to state.

Corollary 2. *Any ECB ϕ of Example* 11 *can be obtained with the following inductive procedure. If $X \in C^1$ then let $\phi(X) = (X, X, \emptyset)$. If $\phi(S)$ has been defined for all $S \in C^n$ for all $n < k$ then for $X \in C^k$ let*

$$M(X) = \{Z \in C_X \setminus \{X\} | Z \in W_X \text{ and } \phi(Z) = (Z, Z, \emptyset)\}$$

Let $\phi(X) = (X, X, \emptyset)$ if $M(X) = \emptyset$, otherwise let $\phi(X) = (X, T, T')$ for some $T \in \arg\min_{A \in M(X)} \gamma_A$, where T' is any coalition that satisfies $T' \subseteq T$ and $T' \in \mathcal{W}_X$. Proceeding inductively ϕ is defined for the whole game.

The characterization provided above is exactly the same as the characterization provided by Acemoglu, Egorov and Sonin (2008).

Many political situations are similar to the ones studied here in the sense that the negotiation process and the rules with which coalitions form is not well defined, but the general structure of the game is defined. In such situations ECB can be used quite naturally without the need to complement the description of the game with the rules of coalition formation. Furthermore, as the solutions to Examples 11 and 13 point to, it should be much easier to use ECB to solve these problems than to define the exact rules of negotiation and using a noncooperative solution concept.

7.2 Finite Acyclic Games

Another class of games for which we can find an algorithm that characterizes the ECBs is finite acyclic games. The following examples demonstrate some situations that can be modeled as a finite acyclic game.

Example 14. ***(Finite Extensive Form Games)*** *Any finite extensive form game of perfect information.*

Example 15. ***(Cournot Oligopoly and Binding Agreements)*** *There are n identical firms producing output at a fixed cost c. The linear market demand is given by $p = A - by$, where y is the aggregate output, A and b are constants. At the start of the game any group of firms S can bindingly come together and form a cartel. Once S is formed, no firm can leave or join this cartel. Following this, any group of firms in $N \setminus S$ can bindingly form a cartel. This continues until a partition P of N forms.*

Utilities are defined over partitions of N. Each cartel plays noncooperatively with other cartels, and share the profits equally among its members. That is, the payoff of $i \in S \in P$ is $\frac{(A-c)^2}{b|S|(|P|+1)^2}$, where $|P|$ is the number of coalitions in partition P. This game has been analyzed by Bloch (1996) and Ray and Vohra (1999).

Figure 11 demonstrates the game for $n = 3$. Note that, 'no action' is not a possible action at any node, therefore the game moves onto terminal nodes. Both Bloch (1996) and Ray and Vohra (1999) make the assumption that each firm can commit to remain alone, I make the same assumption.

Example 16. ***(Network Formation)*** *N is the finite set of players. A network is an undirected graph g with the node set N. Let G be the set of all networks. Each individual has a utility defined over G. The game starts at the empty network. Any two players might agree to form links between themselves, once the link is formed any other two players can form a link. This goes on until a stable network emerges. Once a link is formed, it cannot be severed. Aumann and Myerson (1988) study this game.*

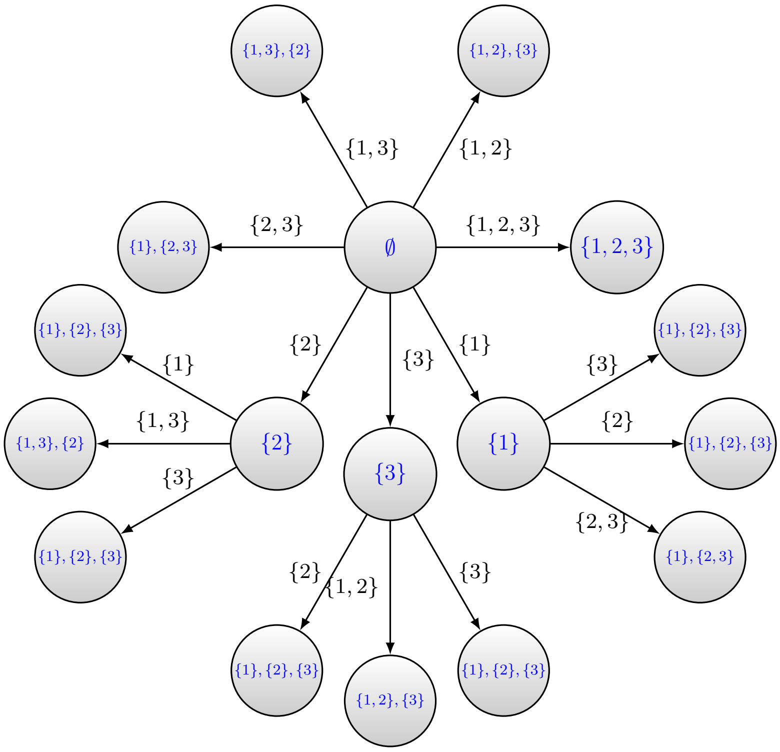


Figure 11: Cournot Oligopoly

In these games, one step deviation property of the ECB holds. This is because any profitable deviation leads to a finite acyclic path and hence can be replaced with a profitable one step deviation by Proposition 11. This implies that we can find all of the ECBs by working backwards. Now, I will formalize this.

Let Z_0 denote the set of exogenously stable nodes of Γ, i.e. $z \in Z_0$ iff $A_z = \{z, z, \emptyset\}$. Let $Z_1 \subseteq Z \setminus Z_0$ denote the set of nodes from which only nodes in Z_0 are reachable. Suppose Z_k is defined for all $k \leq n$ then let $Z_{n+1} \subseteq Z \setminus \{\cup_{i=1,..,n} Z_i\}$ be the set of nodes from which only the nodes in $\{\cup_{i=1,..,n} Z_i\}$ are reachable. Stop doing this when all nodes are exhausted, lets denote the last nonempty set Z_K, since the game is finite we will eventually stop.

$Z_0, Z_1, .., Z_K$ is partition of Z such that no $z \in Z_i$ is reachable from any $z \in Z_j$ where $j < i$. The following proposition provides an algorithm that characterizes the ECBs of a finite acyclic game. For any $z \in Z$, let $N(z)$ denote the nodes adjacent to z, i.e. $N(z) = \{z' \in Z | (z, z', S) \in A_z$ for some $S \subseteq N$, where $S \neq \emptyset\}$.

Proposition 9. *ϕ is an ECB of the finite acyclic game Γ iff ϕ can be obtained with the following procedure.*

For any $z \in Z_0$, let $\phi(z) = (z, z, \emptyset)$. Suppose ϕ is defined for all $z \in Z_n$ for $n < k$. Then for $x \in Z_k$ define

$$\mu(x) = \{z \in N(x) | \{(x, z, S), \sigma(z)\} \succ_S (x, x, \emptyset) \textit{ for some } S \subseteq N\}$$

and

$$\mu'(x) = \{z \in N(x) | \{(x, z, S), \sigma(z)\} \succeq_S (x, x, \emptyset) \textit{ for some } S \subseteq N\}^{29}$$

if $(x, x, \emptyset) \in A_x$, otherwise let $\mu(x) = \mu'(x) = N(x)$.

[29] $x \succeq_S y$ if $x \succeq_i y$ for all $i \in S$ and $x \succ_S y$ if $x \succ_i y$ for all $i \in S$.

Let $\phi(x) = (x, x, \emptyset)$ if $\mu'(x) = \emptyset$. Otherwise let $\phi(x) = (x, z, S)$ for some z, S. Where (i) if $z = x$ then $\mu(x) = \emptyset$, (ii) if $z \neq x$ then $z \in \mu'(x)$, (iii) $\{(x, z, S), \sigma(z)\} \succeq_S (x, x, \emptyset)$ if $(x, x, \emptyset) \in A_x$ and (iv) there does not exist $z' \in Z$, $T \subseteq N$, $i \in S$ with $(x, z', T) \in A_x$ and $\{(x, z', T), \sigma(z')\} \succ_{T \cup i} \{(x, z, S), \sigma(z)\}$.

Note that the algorithm reduces down to backward induction in finite extensive form games. In more general finite acyclic games the idea is the same, we start at the exogenously stable nodes and then we move our way up. Finally, we can use the algorithm to find the ECBs of the Cournot game in Example 15.

Cournot Oligopoly with Binding Agreements

Here, I will solve the Cournot game in Example 15. The problem and the extended coalitional game corresponding to the problem is described above. Any $z \in Z$ is a partition of some $S \subseteq N$, where each coalition in the partition denotes a binding agreement to form the corresponding cartel. Let $|z|$ denote the number of agreements in z and let $N \setminus z$ be the set of players who have not made an agreement yet.

Note that each node for which $|N \setminus z| = 0$ or $|N \setminus z| = 1$ is a terminal node as the whole partition of N is determined in such nodes. So, we start at the nodes z for which $|N \setminus z| = 2$, as only terminal nodes are reachable from these. For any such node the only possible actions are either a player will declare that it will remain isolated or the two remaining players will form a cartel. By the symmetry of the game, the action that maximizes the payoff of one of the remaining players will be taken. Once we assign a prediction to each such node, we can move on to any node z for which $|N \setminus z| = 3$. We can continue this until we reach the root of the game, i.e. the node z^* for which $|N \setminus z^*| = n$, at which point the ECB will be defined. The following proposition pins down the prediction of ECBs in this example, for details see the proof in the Appendix.

Proposition 10. *Let Γ be a Cournot game corresponding to Example 15, then an ECB exists for Γ and if P^* is the predicted outcome then $P^* = \{S^*, \{i\}_{i \notin S^*}\}$ where $|S^*|$ is the first integer following $(2n + 3 - \sqrt{4n + 5})/2$. (If $\sqrt{4n + 5}$ is an integer then $|S^*|$ can take two values, $(2n + 3 - \sqrt{4n + 5})/2$ and $(2n + 5 - \sqrt{4n + 5})/2$)*

The characterization turns out to be equivalent to the characterization in Bloch (1996) and Ray and Vohra (1999). It is important to note that both Bloch (1996) and Ray and Vohra (1999) needed to complement the description of the game with the rules of negotiation and coalition formation. Whereas ECB is directly applied to the description of the game, completely abstracting away from how agreements are reached.

8 Concluding Remarks

8.1 One Step Deviation Property

The deviations from a coalitional behavior can be very complicated, hence it is important to know the conditions under which the existence of a profitable deviation implies the existence of a simple profitable deviation. The following proposition provides such a condition.

Proposition 11. *Let ϕ be a coalitional behavior and suppose there exists a profitable deviation by S to ϕ'. Then the deviation can be replaced with a profitable one step deviation* *if there exists $z^* \in Z$ with $\phi(z^*) \neq \phi'(z^*)$ such that $\sigma'(z^*)$ is not a cycle and there are finitely many $z \in \sigma'(z^*)$ such that $\phi(z) \neq \phi'(z)$.*

The following corollary directly follows from the proposition.

Corollary 3. *In any finite horizon acyclic game ECB satisfies the one step deviation property.*

If the game is not finite or if the game is cyclic then the one step deviation property need not hold. The example of the infinitely repeated favor exchange in Section 4.2 shows that acyclic infinite horizon games may fail to satisfy the one step deviation property. Whereas the example in Figure 8 shows that a finite cyclic game may fail to satisfy the one step deviation property.

Finally, in some games it is easy to see that there exists no profitable deviation that leads to a cycle or an infinite path of play, examples are Rubinstein's bargaining game and beeline games. Directly from Proposition 11 it is easy to see that in these games one step deviation property holds.

Corollary 4. *Suppose Γ is such that for any $i \in N$, $z \in Z$ and $h \in H_z$, we have that $h \succeq_i h'$ whenever h' is a cycle or an infinite path. Then Γ satisfies the one step deviation property.*

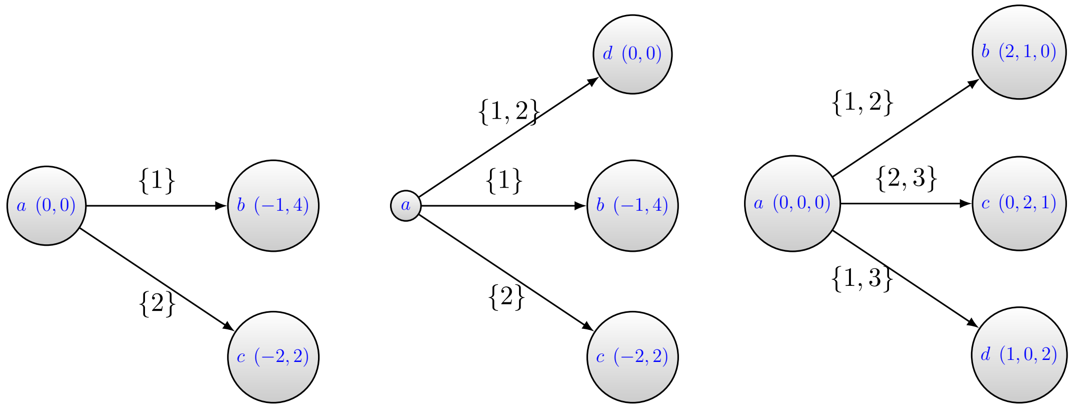
8.2 Existence

The generality of an extended coalitional game makes it hard to provide a general existence result. Nevertheless, the paper contains some indirect existence results in certain classes of games. For instance, an ECB exists in balanced characteristic function games and finite extensive form games.

Figure 12a taken from Chwe (1994) provides an example of a simple game with no ECB. In this example in no ECB we can have $\phi(a) = (a, a, \emptyset)$ since player 2 has an incentive to deviate and move to c. But the coalitional behavior in which player 2 moves from a to c is not an ECB either, because players 1 and 2 can together induce the coalitional behavior in which player 1 moves instead of 2 and this is a profitable deviation for both. But this is also not an ECB, because in this case player 1 would like to deviate by refusing to move. We have returned back to where we started. So, there exists no ECB in this game.

Chwe (1994) calls this example 'preemption', that we would expect player 1 to take a detrimental action to his welfare to preempt player 2 from taking an action that is even worse. Nevertheless, we see that player 1 taking his action is not an ECB, because he falsely believes that he can deviate to 'no action', whereas the consent of 2 is needed for such a deviation. That is why I think the game in Figure 12b captures the story behind 'preemption' more accurately and there is a unique ECB in that game in which 1 preempts 2 by taking the action available to him. Note that the difference is that in Figure 12b, 'no action' is explicitly included as an arc in the graph and the set of players that can take that action is the set of players needed to stop any other action to be taken.

Another example in which an ECB does not exist is represented in Figure 12c. We see that there is a cycle among all possible coalitional behaviors of this game, which precludes existence. Note that, although the root is taken as a state, unlike the above example this has no consequence on the existence problem.



(a) Preemption 1 (b) Preemption 2 (c) Nonexistence

9 Appendix

Proof of Lemma 1. Directly follows from Corollary 3 in Section 8.1.

Proof of Proposition 1. Since ECB satisfies the one step deviation property, any profitable coalitional deviation can be replaced by a profitable one step deviation. Since each subgame perfect equilibrium is immune to profitable one step deviations, we have that each subgame perfect equilibrium is an ECB. Each ECB is a subgame perfect equilibrium by Lemma 2.

Proof of Lemma 2. Let ϕ be a coalitional behavior that is not a subgame perfect equilibrium. Then there exists an individual $i \in N$ that can deviate to a coalitional behavior ϕ' such that $\sigma'(z^*) \succ_i \sigma(z^*)$ for some $z^* \in Z$. Let $Z_1 = \{z \in \sigma'(z^*) | \phi(z) \neq \phi'(z)\}$ and let $Z_2 = \{z \in Z_1 | \sigma(z) \succeq_i \sigma'(z)\}$. If $Z_2 = \emptyset$ then consider the deviation from ϕ by i that only includes the actions in Z_1, which is a deviation that increases the payoff of i at every node at which an action changes. If $Z_2 \neq \emptyset$ then let z' be the node in Z_2 that is closest to z^*. Consider the deviation by i from ϕ, that only involves the actions at the nodes in Z_1 that are in between z^* and z' (including z^*, not including z'). The resulting deviation is profitable. Hence, ϕ is not an ECB.

Proof of Proposition 2. If ϕ is an ECB then by Lemma 2, ϕ is a subgame perfect equilibrium. Furthermore since it is an ECB, it is also immune to coalitional deviations. This proves the only if part.

Now suppose ϕ is a subgame perfect equilibrium that is immune to coalitional deviations as described in the proposition. But then by Proposition 11 in Section 8.1, any profitable deviation can be replaced by a profitable one step deviation. But each one step deviation is an individual deviation. Since ϕ is a subgame perfect equilibrium, this implies that there does not exist a profitable deviation and hence ϕ is an ECB.

Proof of Lemma 3. Let ϕ be an ECB under myopia.

First suppose that (x, P) is stable, but is not in the core. Then there exists a coalition S and $y_S \in V(S)$ such that $y_S > x_S$. By the conditions on the set of actions, S can deviate and induce a path with the first outcome being some (z, P'), where $z_S = y_S$. The only state at which an action changes is (x, P) and S is better off here, hence the deviation is profitable. A contradiction.

Now assume that $(x, P) \in C(N, V)$ and $(x, P) \notin S(\phi)$. Suppose $\phi((x, P)) = ((x, P), (x', P'), T)$. If for some $j \in T$ we have $x_j > x'_j$ then there exists a profitable deviation in which j blocks the move, since ϕ is an ECB this would be a contradiction. But then we have $x'_i \geq x_i$ for all $i \in T$. Furthermore if $x'_i > x_i$ for all $i \in T$ then $(x, P) \notin C(N, V)$, therefore we must have $x'_i \geq x_i$ for all $i \in T$ and $x'_k = x_k$ for some $k \in T$.

Proof of Proposition 3. The construction used in the proof is similar to the ones found in Diamantoudi and Xue (2003) and Konishi and Ray (2003).
1st Step: If $(x^*, P^*) \in C(N, V)$ then there exists an ECB under foresight ϕ such that $\mathcal{T}(\sigma((x, P))) = (x^*, P^*)$ for all $(x, P) \in Z$.

The proof is by construction, I will construct an ECB under foresight ϕ with the desired property.

Take any $(x^*, P^*) = (x^*, \{S_1^*, S_2^*, .., S_K^*\}) \in C(N, V)$. For any coalition S, let $\overline{S}$ denote the partition of S composed of singletons. Let $\phi(x^*, P^*) = ((x^*, P^*), (x^*, P^*), \emptyset)$.

Let $(x_t, P_t) = (x_t, \{S_1^*, ..., S_t^*, \overline{\cup_{j=t+1,..,K} S_j^*}\})$, where $x_t(S_j^*) = x^*(S_j^*)$ for all $j = 1, .., t$ and $x_t(\{i\}) = \max v(\{i\})$ for any $i \in \cup_{j=t+1,..,K} S_j^*$. For any $t = 0, 1, .., K-1$, let $\phi(x_t, P_t) = ((x_t, P_t), (x_{t+1}, P_{t+1}), S_{t+1}^*)$.

For any $(x, P) \neq (x_t, P_t)$ for any $t = 0, 1, .., K$, let $\phi((x, P)) = ((x, P), (x', P'), \{i\})$, where $x'(\{i\}) = \max v(\{i\})$, $\{i\} \in P'$ and where i is the player with the smallest index for which $x_i^* \geq x_i$ and $i \in S \in P$, where $|S| \geq 2$. Note that since $(x^*, P^*) \in C(N, V)$, such a player exists.

This completes the specification of ϕ, note that ϕ is a coalitional behavior as it assigns a unique action for each $z \in Z$. Furthermore $\mathcal{T}(\sigma(z)) = (x^*, P^*)$ for all $z \in Z$. Now we need to show that ϕ is an ECB. As any deviation leading to an infinite path would lead to a utility of $\max v(\{i\})$, by Proposition 11 we can restrict attention to one step deviations.

As any one step deviation at (x^*, P^*) leads to a cycle there exists no profitable one step deviation at (x^*, P^*). Assume that there is a profitable one step deviation at some $(x, P) \neq (x^*, P^*)$, but if the deviating coalition is taking another action at (x, P) then the deviation is not profitable since it will again end up at (x^*, P^*). Then the deviating coalition is inducing 'no action' at (x, P), but since the coalition moving at (x, P) weakly prefers (x^*, P^*) to (x, P), this cannot be a profitable deviation. Contradiction. Hence, ϕ is an ECB.
2nd Step: Suppose ϕ is an ECB under foresight such that $\mathcal{T}(\sigma((x, P))) = (x^*, P^*)$ for all $(x, P) \in Z$ then $(x^*, P^*) \in C(N, V)$.

Towards a contradiction suppose ϕ is an ECB under foresight such that $\mathcal{T}(\sigma(x, P)) = (x^*, P^*)$ for every $(x, P) \in Z$ but (x^*, P^*) is not in the core. Then there exists (x, P) such that $S \in P$ and $x_S > x^*(S)$. But ϕ induces a finite path from (x, P) to (x^*, P^*) and at some point someone in S is active on this path. Let (x', P') be the first node on the path for which some $j \in S$ is active, let j deviate by refusing to take the action. The resulting deviation is profitable as j is getting $x'_j = x_j$ instead of x_j^*. A contradiction.

Proof of Proposition 4. Towards a contradiction suppose ϕ is an ECB such that $\mathcal{T}(\sigma(x, P)) = (x^*, P^*)$ for every $(x, P) \in Z$ but (x^*, P^*) is not in the core. Then there exists (x, P) such that $S \in P$ and $x_S > x^*(S)$. But ϕ induces a finite path from (x, P) to (x^*, P^*) and at some point someone in S is active on this path. Let (x', P') be the first node on the path for which some $j \in S$ is active, let j deviate by refusing to take the action. The resulting deviation is profitable as j will ultimately be getting $x'_j = x_j$ instead of x_j^* and $\delta \to 1$.

Proof of Proposition 5. Let ϕ be an FECB. I will use the function used by Chwe (1994) in his proof of Proposition 1 of his paper. Define $f : 2^Z \to 2^Z$ as $f(X) = \{a \in Z|$ for all d, S such that $(a, d, S) \in A_a$ there exists $e \in X$ where $d = e$ or $e \gg d$ such that, $v_i(a) \geq v_i(e)$ for some $i \in S\}$. Chwe showed that LCS contains all sets X for which $f(X) \supseteq X$. Hence, if we can show that $S(\phi) \subseteq f(S(\phi))$ then we are done.

Towards a contradiction assume not. Then there exists $x \in S(\phi)$ such that there exists d, S with $(x, d, S) \in A_x$ and for all $e \in S(\phi)$ where $e = d$ or $e \gg d$ we have $v_i(e) > v_i(x)$ for all $i \in S$. First suppose $d \in S(\phi)$, but then since $v_i(d) > v_i(x)$ for all $i \in S$, we have that S has a profitable deviation from ϕ. A contradiction.

Now suppose that $d \notin S(\phi)$. Then $\mathcal{T}(\sigma(d)) \in S(\phi)$ and $\mathcal{T}(\sigma(d)) \gg d$, because otherwise there is a profitable deviation in which the coalition who is not better off can deviate by refusing to move. But then $v_i(\mathcal{T}(\sigma(d))) > v_i(x)$ for all $i \in S$ and moving to d from x is a profitable deviation for S, hence ϕ is not an ECB. A contradiction.

Proof of Proposition 6. Take Γ that satisfies NI and MOA.

First suppose that V is an SREFS and ϕ is the coalitional behavior that supports it. I will show that ϕ is an FECB. First note that by Proposition 11 in Section 8.1 FECB satisfies the one step deviation property, hence it suffices to check one step deviations. Suppose there is a profitable deviation at some unstable x in which some i blocks the move, but then $v_i(x) > v_i(\mathcal{T}(\sigma(x)))$, which is a contradiction to ES. Suppose there is a profitable deviation at some unstable x where i blocks an action leading to z and T takes an action leading to y. By MOA there is also a profitable deviation in which i blocks the action leading to z and $T \cup \{i\}$ takes an action leading to y. Furthermore since the deviation is profitable we have $v_j(\mathcal{T}(\sigma(y))) > v_j(\mathcal{T}(\sigma(x)))$ for all $j \in \{T \cup i\}$, which is a contradiction to M. Finally suppose there is a profitable deviation at some stable outcome x by coalition S to an outcome z, but then $v_j(\mathcal{T}(\sigma(z))) > v_j(\mathcal{T}(\sigma(x)))$ for all $j \in S$, a contradiction to IS. But we have exhausted all possible one step deviations, hence ϕ is an FECB.

Now suppose that ϕ is an FECB. We will show that $S(\phi)$ is an SREFS supported by ϕ. Note that at any state x with $\phi(x) = (x, x, \emptyset)$ we have that there does not exist y and S such that $(x, y, S) \in A_x$ and $v_i(\mathcal{T}(\sigma(y))) > v_i(\mathcal{T}(\sigma(x)))$ for all $i \in S$, as otherwise S has a profitable deviation. Hence IS is satisfied.Now take any state x for which $\phi(x) = (x, y, S)$ for some y and S. First note that $v_j(\mathcal{T}(\sigma(x))) > v_j(x)$ for all $j \in S$, since otherwise by NI there exists $i \in S$ for which $v_i(x) > v_i(\mathcal{T}(\sigma(x)))$ in which case i has a profitable deviation at x. But then ES is satisfied. Finally if M is violated then trivially there exists a profitable deviation and ϕ is not an FECB, so M is also satisfied.

Proof of Proposition 7. Figure 8 shows that an EPCF may not be a DECB, so here I will only show that each DECB is an EPCF. Let ϕ be a DECB. First note that for any x, y, S, where $\phi(x) = (x, y, S)$, a straightforward calculation shows that $\sigma(y) \succ_S \sigma(x)$ iff $\sigma(x) \succ_S (x, x, \emptyset)$ (Remember that $(x, x, \emptyset)$ is the path of play corresponding to staying at x and utility of a path corresponds to the discounted utility of the states visited along the path). I will refer to this observation as $(*)$.

Take any $x \in Z$ with $\phi(x) = (x, y, S)$ where $x \neq y$, first observe that $\sigma(x) \succ_S (x, x, \emptyset)$, as otherwise by NI there exists $i \in S$ for which $(x, x, \emptyset) \succ_i \sigma(x)$, who would have a profitable deviation in which she blocks the taken action. By $(*)$ this implies that $\sigma(y) \succ_S \sigma(x)$. Towards a contradiction assume that there exists z with $(x, z, S) \in A_x$ and $\sigma(z) \succ_S \sigma(y)$. But then there exists a profitable deviation by S to z, a contradiction. So, ϕ satisfies the first condition.

Now assume that x is such that there exists y, S with $(x, y, S) \in A_x$ and $\sigma(y) \succ_S \sigma(x)$. Towards a contradiction assume $\phi(x) = (x, x, \emptyset)$. But then $\sigma(y) \succ_S (x, x, \emptyset)$, but this implies that $\{(x, y, S), \sigma(y)\} \succ_S (x, x, \emptyset)$. But then there is a profitable deviation by S to y, a contradiction. So, ϕ also satisfies the second condition and it is an EPCF.

Proof of Lemma 4. For the first part see the proof of Proposition 8. For the second part, suppose ϕ is a coalitional behavior such that for some $z \in Z$ we have $L(\sigma(z)) > 1$. Consider the deviation in which the initiator S of $\phi(z)$ changes the action to $(z, \mathcal{T}(\sigma(z)), S)$, since the game is a beeline game S can do this. The deviation is profitable because of the transaction costs, so ϕ is not an ECB.

Proof of Proposition 8. First I will show that the resulting coalitional behavior is an ECB. Note that the game satisfies the one step deviation property by Proposition 11, as any deviation to an infinite length path will provide the worst utility. Hence, I will check one step deviations. Suppose $\phi(z) = (z, z, \emptyset)$, then there exists no stable outcome that dominates z and every outcome that is reachable from z is reachable in one step. Hence, there is no deviation. Suppose $\phi(z) = (z, x, S)$, then x is stable. Futhermore $v_j(x) > v_j(z)$ for all $j \in S$ and hence there is no profitable deviation in which some $i \in S$ refuses to take the action without joining another action. Finally suppose there exists a deviation in which $i \in S$ blocks (z, x, S) and instead T takes the action (z, x', T). Whatever $\mathcal{T}(\sigma(x'))$ is since the game is a beeline game it can be reached in one step from z by T. So, without loss of generality assume $\phi(x') = (x', x', \emptyset)$. But then $v_j(x') > v_j(x)$ for all $j \in \{T \cup i\}$ and x' is stable. A contradiction.

For the other way assume that ϕ is an ECB. As any $z \in Z_0$ is undominated we have that for every $z \in Z_0$, $\phi(z) = (z, z, \emptyset)$. Assume that all the stable outcomes of ϕ can be obtained from the algorithm for all $z \in Z_n$ for all $n < k$. Take any $x \in Z_k$. Suppose $\phi(x) = (x, x, \emptyset)$, then there does not exist any stable outcome that dominates x and hence $\mu(x) = \emptyset$ (note that any outcome that dominates x must be in $\cup_{i<k} Z_i$). Finally assume that $\phi(x) = (x, z, S)$. Since by Lemma 4 in any ECB, the prediction is reached in one step we have that $z \in \mu(x)$. Furthermore since ϕ is an ECB we have $v_j(z) > v_j(x)$ for all $j \in S$, as otherwise because of the transaction costs it is a profitable deviation for some $i \in S$ to refuse to take the action.

Finally since ϕ is an ECB there does not exist T, y with $(x, y, T) \in A_x$, $\phi(y) = (y, y, \emptyset)$, and $i \in S$ such that $v_j(y) > v_j(z)$ for all $j \in \{T \cup i\}$. Otherwise, trivially there is a deviation.

Proof of Corollary 2. Note that any $X \in \mathcal{C}^1$ is undominated and any $X \in \mathcal{C}^k$ can only be dominated by some $Y \in \mathcal{C}^l$, where $l < k$, furthermore for such Y we need to have $Y \subseteq X$. But then directly from the 1st Step in Proposition 8 we have that any $X \in \mathcal{C}^1$ is stable and any $X \in \mathcal{C}^k$ is stable iff it is not dominated by some stable $Y \in C_X \setminus \{X\}$. So, we only need to show the $2nd$ step of Proposition 8.

By Proposition 8 for any X unstable, we have $\phi(X) = (X, T, T')$ for some T', T where (i) T is stable, $v(T) >_{T'} v(X)$ and (ii) there does not exist S' stable, $S \subseteq X$, $j \in T'$ with $(X, S', S) \in A_X$ such that $v(S') >_{S \cup \{j\}} v(T)$. I will show that T', T satisfies (i) and (ii) iff $T \in \arg\min_{A \in M(X)} \gamma_A$ and T' is any coalition that satisfies $T' \in \mathcal{W}_X$ and $T' \subseteq T$.

Note that if $T \in \arg\min_{A \in M(X)} \gamma_A$, $T' \in \mathcal{W}_X$ and $T' \subseteq T$, then T', T trivially satisfy (i). Suppose it violates (ii) but then there exists S' stable, $S \subseteq X$, $j \in T'$ with $(X, S', S) \in A_X$ such that $v(S') >_{S \cup \{j\}} v(T)$. Since $v(S') >_S v(T)$, we have $S \subseteq S'$. Since $S \in W_X$, we have $S' \in W_X$, finally since $j \in T'$ prefers S' to T we have $\gamma_{S'} < \gamma_T$. But then $T \notin \arg\min_{A \in M(X)} \gamma_A$, a contradiction.

Now suppose that T, T' satisfies (i) and (ii). Suppose there exists $F \in M(X)$ with $\gamma_F < \gamma_T$. But then $F \cap T' \neq \emptyset$ (as $\alpha \geq \frac{1}{2}$) and $v_i(F) > v_i(T)$ for all $i \in F \cup \{F \cap T'\}$ (since v_i is increasing in relative power), a contradiction to (ii). So, we only need to show that $T' \subseteq T$. By (i), since $v_i(T) > v_i(X)$ for all $i \in T'$, we have $T' \subseteq T$.

Proof of Proposition 9. First I will show that the resulting coalitional behavior is an ECB. Note that the game satisfies one step deviation property, therefore I will check one step deviations. Suppose $\phi(z) = (z, z, \emptyset)$, then there exists no action that can be taken at z such that the initiator strictly prefers the resulting path. Hence, there is no deviation. Suppose $\phi(z) = (z, x, S)$, where $z \neq x$, then $x \in \mu'(z)$. This means that either $(z, z, \emptyset) \notin A_z$ or $\sigma(z) = \{(z, x, S), \sigma(x)\} \succeq_S (z, z, \emptyset)$, hence there is no profitable deviation in which some $i \in S$ refuses to take the action without joining another action. Finally suppose there exists a deviation in which $i \in S$ blocks (z, x, S) and instead T takes the action (z, x', T). But then $\{(z, x', T), \sigma(x')\} \succ_{T \cup i} \{(z, x, S), \sigma(x)\}$. A contradiction.

For the other way assume that ϕ is an ECB. As any $z \in Z_0$ is exogeneously stable we have that for every $z \in Z_0$, $\phi(z) = (z, z, \emptyset)$. Assume that ϕ can be obtained from the algorithm for every $z \in Z_n$ for $n < k$. Take any $x \in Z_k$. Suppose $\phi(x) = (x, x, \emptyset)$, then there does not exist any $z \in N(x)$ with $\{(x, z, S), \sigma(z)\} \succ_S (x, x, \emptyset)$ for some S (note that any outcome that is reachable from x must be in $\cup_{i<k} Z_i$). Finally assume that $\phi(x) = (x, z, S)$. Since ϕ is an ECB we have that either $(x, x, \emptyset) \notin A_x$ or $\{(x, z, S), \sigma(z)\} \succeq_S (x, x, \emptyset)$, hence $z \in \mu'(x)$. Furthermore since ϕ is an ECB there does not exist T and x' with $\{(z, x', T), \sigma(x')\} \succ_{T \cup i} \{(z, x, S), \sigma(x)\}$. We are done.

Proof of Proposition 10. At any outcome $z \in Z$, let $|z|$ denote the number of agreements in z and let $N \setminus z$ denote the set of players that have not made an agreement yet. Note that the proof is very similar to Bloch (1996)'s proof, that is why I have skipped many of the calculations. To be more precise, when I say 'the optimal move is x', I directly take this observation from Bloch's paper and skip the calculation. Please refer to the paper for details.

1st Step: Show that at any subgame starting at z where $|N \setminus z| < (|z| + 1)^2$ an ECB exists and any ECB specifies that $N \setminus z$ remains alone.

First note that if z is such that $N \setminus z = \{i\}$, then the only possibility is for i to stay alone. Now assume that z is such that $|N \setminus z| < (|z| + 1)^2$ and for any subgame reachable from z ECB specifies that all subsequent players will choose to remain alone. At z the payoff of a player who forms a coalition S is $\frac{(a-c)^2}{|S| b(|N \setminus z| - |S| + |z| + 2)^2)}$, which is maximized at $|S| = 1$ (see Bloch(1996) for the calculation). Which means that if a coalitional behavior requires a coalition S with $|S| > 1$ to move at z, any $j \in S$ can deviate by instead moving by himself and that would be a profitable deviation. Similarly if a coalitional behavior specifies that j is moving alone at z then there exists no profitable deviation since this move maximizes the payoff of j.

2nd Step: Show that at any subgame starting at z where $|N \setminus z| = (|z| + 1)^2$ an ECB exists and it either specifies that some $j \in N \setminus z$ moves alone or $N \setminus z$ forms a coalition at z.

Suppose z is such that $|N \setminus z| = (|z| + 1)^2$. By Step 1 in any subgame reachable from z all the remaining players will decide to remain isolated, so at z the payoff of a player who forms a coalition S is $\frac{(a-c)^2}{|S| b(|N \setminus z| - |S| + |z| + 2)^2}$, which is maximized at $|S| = 1$ and $|S| = |N \setminus z|$. Which implies that only the coalitional behaviors that specify that some $j \in N \setminus z$ moves alone or $N \setminus z$ forms a coalition will be immune to deviation.

3rd Step: Show that at any z where $(|z|+2)^2+1 > |N \setminus z| > (|z|+1)^2$ any ECB specifies that $N \setminus z$ will form a coalition.

Suppose z is such that $(|z|+2)^2+1 > |N \setminus z| > (|z|+1)^2$. By Step 1 in any subgame reachable from z all the remaining players will decide to remain isolated, so at z the payoff of a player who forms a coalition S is $\frac{(a-c)^2}{|S|b(|N\setminus z|-|S|+|z|+2)^2}$, which is maximized at $|S| = |N \setminus z|$.

4th Step: Show that at any z where $(|z|+2)^2+1 = |N \setminus z|$ an ECB either specifies that $N \setminus z$ will form a coalition or some $j \in N \setminus z$ will declare that it will stay alone or $N \setminus z \setminus j$ will form a coalition.

Steps 1 through 3 establish the ECB for every subgame reachable from z. If in any subgame z' where $|N \setminus z'| = (|z'|+1)^2$ the ECB specifies that the remaining players will merge then the optimal move for player $j \notin z$, $j \in z'$ is to remain alone. This means that the coalitional behavior that specifies that j is moving alone at z is an ECB. In this case, if a coalitional behavior specifies that a coalition S including j where $|S| > 1$ moves then j has a profitable deviation. If a coalitional behavior specifies that $|S| > 1$ that does not include j moves then observe that since in the subgame following this every other player will remain isolated this move can only be profitable if $N \setminus z \setminus j$ moves, otherwise there exists a deviation in which S would stop moving but instead let j move alone. Note that both of the described ECBs lead to the same outcome.

If at every subgame z' where $|N \setminus z'| = (|z'|+1)^2$ the ECB specifies that the remaining players remain isolated, then the optimal move at z is for the remaining players to merge. Hence that is the only ECB.

5th Step: Show that at any z where $(|z|+2)^2+1 < |N \setminus z|$ any ECB specifies that some $j \in |N \setminus S|$ will declare that it will stay alone.

This step is solved with induction. First think about a z such that $(|z|+2)^2+1 < |N \setminus z|$ and at any subgame reachable from z ECB is defined in Steps 1 through 4. Any player $j \in N \setminus S$ would maximize its payoff by moving alone, so ECB has to specify that some j will move alone. Once we are done with this we can move up the tree to some other z', at which point the ECB for every subgame that is reachable from z' is defined. But at each such step we will have that it is optimal for $j \in N \setminus z'$ to make the move alone.

6th Step: The result.

Steps 1 through 5 establish that the ECB outcomes will be equivalent to the outcomes of the game Bloch considers. Then the proposition follows from Bloch's calculations.

Proof of Proposition 11. Assume S has a profitable deviation from ϕ to ϕ', let z^* be such that $\phi(z^*) \neq \phi'(z^*)$, $\sigma'(z^*)$ is not a cycle and there are finitely many $z \in \sigma'(z^*)$ such that $\phi(z) \neq \phi'(z)$. Then there exists $z' \in \sigma'(z^*)$ such that $\phi(z') \neq \phi'(z')$, but $\phi(z) = \phi'(z)$ for all $z \in \sigma'(z')$ such that $z \neq z'$. Think about the deviation in which S changes $\phi(z')$ to $\phi'(z')$, this is a one step deviation and furthermore z' is the only node for which the action changes and it changes from $\phi(z')$ to $\phi'(z')$. Since the initial deviation is profitable we have $\sigma'(z') \succ_S \sigma(z')$, hence this one step deviation is also profitable.

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