Mechanisms with Referrals: VCG Mechanisms and Multilevel Mechanisms*

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Abstract

We study mechanisms for environments in which only some of the agents are directly connected to a mechanism designer and the other agents can participate in a mechanism only through the connected agents' referrals. In such environments, the mechanism designer and agents may have different interest in varying participants so that agents strategically manipulate their preference as well as their network connection to avoid competition or congestion; while the mechanism designer wants to elicit the agents' private information about both preferences and network connections.

As a benchmark for an efficient mechanism, we re-define a *VCG mechanism*. It is incentive compatible and individually rational, but it generically runs a deficit as it requires too much compensation for referrals. Alternatively as a budget-surplus mechanism, we introduce a *multilevel mechanism*, in which each agent is compensated by the agents who would not be able to participate without her referrals. Under a multilevel mechanism, we show that fully referring one's acquaintances is a dominant strategy and agents have no incentive to under-report their preference if the social welfare is submodular.

keywords mechanism design; referral program; reward scheme; VCG mechanism; multilevel mechanism; incentive compatibility; budget balancedness

JEL classification: D82, D71, C72

1 Introduction

In many social choice problems, the feasible allocations depend on who participates. As the participants change, the desirable allocations also change accordingly and each participant may have different interest in others' participation. Thus individuals strategically interact with each other by letting other potential participants join in or by preventing them from participating, and hence desirable outcomes may be hindered.

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To analyze conflict of interests in varying participants, we adopt a mechanism design approach to an environment in which agents' network connections are their private information. The environment with *incomplete network information*, which we consider, have the following features:

i) A mechanism designer, or a planner, is directly connected to only some of the agents.

ii) The agents who are not directly connected to the designer can participate in the mechanism only through a connected chain of others' *referral*s.

iii) Feasible allocations that the designer can choose depend on the actual participants induced by referrals.

iv) Each agent has two-folded private information; a *preference* over the feasible allocations, and a *network connection* which captures the agent's acquaintances who can be referred.

While taking preference as private information is standard in the mechanism design literature, the novel part of this paper is considering agents' network connection as another dimension of private information. By private information about network connection, we mean that the designer and the agents know about who are directly connected to themselves, but do not know who are connected to others. Thus, agents may strategically conceal their network connection particularly when congestion or competition adversely impacts on the agents.

As a benchmark for an efficient mechanism, we first study a *VCG mechanism*, initially introduced by Vickrey (1961), Clarke (1971), and Groves (1973). While a VCG mechanism is incentive compatible and individually rational; it turns out a VGC mechanism generically runs a deficit when the participants are endogenously determined by referrals. We characterize the necessary and sufficient condition for a VCG mechanism to yield a non-negative surplus to the mechanism designer. Roughly speaking, if agents' marginal contribution to the society by referring others is greater than a certain level, then the mechanism designer has to provide too much compensation to the agents who contribute to other agents' participation and the compensation dominates the agents' total payments. We also provide an alternative condition for budget balancedness: if the core of the *network-restricted per-capita game*[1] is nonempty, then a VCG mechanism runs a deficit.

It is well-known, due to Green and Laffont (1977), that no efficient mechanism is incentive compatible, individually rational, and budget surplus at the same time. In order to overcome the budget problem of a VCG mechanism, therefore, we propose an alternative mechanism, namely a *multilevel mechanism*. Under this mechanism, each

[1]We define this later in Section 3 based on Myerson (1977).

agent's payment is based not only on his own marginal contribution but also on that of other agents who are referred to the mechanism designer through himself; and he receives some compensation from the referred agents. We show that fully referring one's acquaintances is a dominant strategy and agents have no incentive to under-report their preference if the social welfare function is *submodular*. Although a multilevel mechanism is not fully incentive compatible and agents may over-report ther preference, it boosts competitions among the agents and it is good for the designer in terms of her budget. In addition, we characterize agents' beliefs in which truthfully reporting their preference is their best response.

With a supermodular welfare function, agents may spontaneously refer other agents without any explicit reward because an agent's marginal contribution increases as population grows. As Sprumont (1990) shows, if a social welfare function is supermodular, a *population monotonic allocation scheme* exists so that the planner can make all the existing agents happy with new agents' participation. However, there is no such allocation rule if the welfare function is submodular in which more participants causes severe congestion and competition. Our main contribution is to find that multilevel mechanisms have good incentive properties even in such environments with conflict of interests in varying population.

Referral programs are prevalent phenomena in various markets and they have been widely studied in the literature of marketing. Due to recent development of internet technology and social media, studying allocation problems in networks is a popular research area in economics and computer science. Since Megiddo (1978), many papers, including Ågotnes et al. (2009) and Bachrach and Rosenschein (2007), adopt cooperative game solution concepts to allocation problems in networks and analyze incentive problems. In particular, Hougaard and Tvede (2012) and Hougaard and Tvede (2015) study incentive compatibility of allocation mechanisms in minimum cost spanning networks. In those models, referring other agents is not a strategic consideration because a network is fixed and its structure is commonly known.

Another strand of literature studies incentives for referrals. Lee and Driessen (2012) study agents' referral incentives in a variable population environment, proposing a new cooperative solution, namely *sequentially two-leveled egalitarianism*, and comparing it to *Shapley value*. Recently computer science literature, such as Yu and Singh (2003), Singh et al. (2011), Emek et al. (2011), and Drucker and Fleischer (2012), deals with multilevel mechanisms in which small rewards are allocated to mitigate free-riding problems or Sybil attacks in peer-to-peer systems. However, those approaches study incentives given a simple reward scheme, rather than fully analyzing direct mechanisms.

The paper is organized as follows. Section 2 describes a model with asymmetric information about networks in which agents can strategically refer others. In Section 3, we define a VCG mechanism in an environment with varying population, as a benchmark for an efficient mechanism, and characterize its properties related to incentive compatibility, individual rationality, and budget balancedness. In Section 4, we propose a multilevel mechanism, which guarantees a budget surplus, and characterize conditions for incentive compatibility and individual rationality. Section 5 concludes this paper with a remark for future research.

2 A model

Let $N = \{1, 2, \cdots, n\}$ be a set of agents and 0 be a mechanism designer (or a planner). For any $S \subseteq N$, $\Gamma(S)$ is a set of feasible allocations with the participation of S together with 0. Let $\Gamma = \cup_{S \subseteq N} \Gamma(S)$ be a set of all allocations. We assume *free exclusion*, which requires $S' \subseteq S \implies \Gamma(S') \subseteq \Gamma(S)$. Each agent $i \in N$ has a *preference* over the allocations, which is represented by a utility function $v_i : \Gamma \to \mathbb{R}$. For any preference profile $v \equiv \{v_i\}_{i \in N}$ and any coalition $S \subseteq N$, a *social welfare* is defined by

$$W(v, S) = \max_{\gamma \in \Gamma(S)} \sum_{i \in S} v_i(\gamma),$$

and a set of *efficient allocations* is:

$$\Gamma^*(v, S) = \left\{ \gamma \in \Gamma(S) \;\middle|\; \sum_{i \in S} v_i(\gamma) = W(v, S) \right\}.$$

Each agent $i \in N$ has a set $e_i \subseteq N \backslash \{i\}$ of *acquaintances* or i's *connection*, so that i can refer j if $j \in e_i$. The planner 0 also has a set $e_0 \subseteq N$ of agents who are directly connected to her. Note that a connection profile $\{e_i\}_{i \in N \cup \{0\}}$ constitutes a directed network on $N \cup \{0\}$.

For each agent $i \in N$, his preference v_i and his connection e_i are his private information. The mechanism designer, in practice, may sequentially approach agents based on referrals in various ways. Due to the *revelation principal* (Myerson, 1981), however, without loss of generality, we focus on a class of *direct mechanisms*, in which agents are simultaneously asked to report their preference and connection. An agent whose private information is (v_i, e_i) may report any (v'_i, e'_i) with a restriction of $e'_i \subseteq e_i$.

Given a reported (or referred) connection profile e, the set of *participants* $N(e)$ is determined endogenously:

$$N(e) = \{0\} \cup e_0 \cup [\cup_{i \in e_0} e_i] \cup \left[\cup_{i \in [\cup_{j \in e_0} e_j]} e_i\right] \cup \cdots .$$

A connection profile e is *trivial* if $N(e) = e_0 = N$. In a trivial connection profile, a mechanism designer is directly connected to all the agents and hence any referral does not affect either the participants or the set of feasible allocations.

Let $V \times E$ be a profile space. A *mechanism* (g, P) is a pair of an *allocation rule* $g : V \times E \to \Gamma$ such that $g(v, e) \in \Gamma(N(e))$; and a *payment rule* $P : V \times E \to \mathbb{R}^n$ such that $i \notin N(e) \implies P_i(v, e) = 0$ An allocation rule g is *efficient* if, for all $(v, e) \in V \times E$, $g(v, e) \in \Gamma^*(v, N(e))$. Under a mechanism (g, P), given others' reports v_{-i} and e_{-i}, if agent i with the preference of v_i reports v'_i and e'_i, then he gets the payoff $v_i(g(v'_i, v_{-i}, e'_i, e_{-i})) - P_i(v'_i, v_{-i}, e'_i, e_{-i})$. When there is no danger of confusion, we denote $v' = (v'_i, v_{-i})$ whereas $v = (v_i, v_{-i})$, and similarly for e' and e.

A mechanism (g, P) is *incentive compatible* if, for all $(v, e) \in V \times E$, all $i \in N$, all $v'_i \in V_i$, and all $e'_i \subseteq e_i$,

$$v_i(g(v, e)) - P_i(v, e) \geq v_i(g(v', e')) - P_i(v', e').$$

An agent's utility from not participating is normalized to zero. Hence, a mechanism (g, P) is *individually rational* if, for all $(v, e) \in V \times E$ and all $i \in N$,

$$v_i(g(\sigma(v, e))) - P_i(\sigma(v, e)) \geq 0,$$

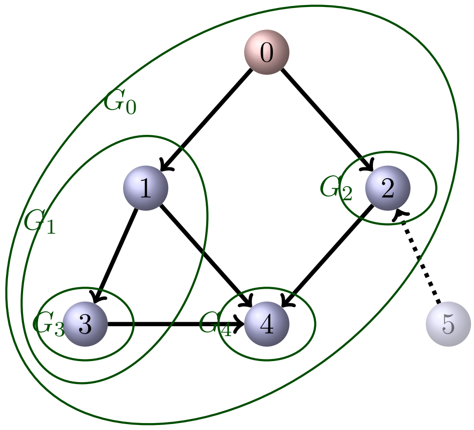
where $\sigma(v, e)$ is an equilibrium strategy profile given (v, e). Given a mechanism (g, P) and (v, e), the sum of agents' payments $\sum_{i \in N} P_i(\sigma(v, e))$ is the *ex-post surplus* to the mechanism designer. A mechanism (g, P) runs a *deficit* for (v, e), if the ex-post surplus to the mechanism designer is negative.

Given $e \in E$ and $i \in N$, the set of i's *outsiders*, $O_i(e)$, consists of agents who can be involved without i, that is $O_i(e) \equiv N(e_{-i}) \setminus \{i\}$. Note that e is trivial if and only if $O_i(e) = N(e) \setminus \{i\}$ for all $i \in N$. The agent i's *group*, $G_i(e)$, consists of agents who can not participate without i, that is $G_i(e) \equiv N(e) \setminus O_i(e)$. Note that e is trivial if and only if $G_i(e) = \{i\}$ for all $i \in N$. As $G = \{G_i(e)\}_{i \in N_0}$ forms a partial ordering, a connection profile e induces a tree. Given $e \in E$, i is a *predecessor* of j if $j \in G_i(e)$. Note that $N(e)$ endowed with this precedence relation is a tree and we call it a *contribution tree*. For all $i \in N$, the set of i's *predecessors* is $R_i(e) \equiv \{j \in N \mid i \in G_j(e)\}$ and the set of i's *immediate followers* is $F_i(e) \equiv \{j \in N \mid G_i(e) \supset G_j(e) \text{ and } (\nexists k \in N \setminus \{i, j\})\ G_i(e) \supset G_k(e) \supset G_j(e)\}$. Figure 1 shows how the contributions to new participants are recognized.

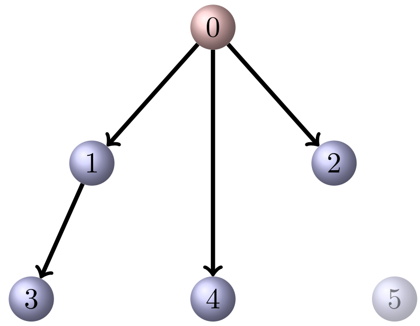
3 A VCG Mechanism

As a benchmark for an efficient mechanism, we re-define a VCG mechanism under incomplete network information. A VCG mechanism consists of an efficient allocation rule g and a payment rule P^V which captures each agent's marginal effect on the remaining agents. Formally, a VCG payment rule P^V is:

$$\begin{aligned} P_i^V(v, e) &= \min_{v'_i, e'_i} W(v'_i, v_{-i}, N(e'_i, e'_{-i})) \quad - \sum_{j \in N(e) \setminus \{i\}} v_j(g(v, e)) \\ &= W(v, O_i(e)) - \sum_{j \in N(e) \setminus \{i\}} v_j(g(v, e)), \end{aligned}$$



(a) A connection profile



(b) The induced contribution tree

Figure 1: A connection profile and the induced contribution tree.
Let $N = \{1,2,3,4,5\}$ and $e = \{e_i\}_{i \in N_0}$ with $e_0 = \{1,2\}$, $e_1 = \{3,4\}$, $e_2 = \{4\}$, $e_3 = \{4\}$, $e_4 = \emptyset$, and $e_5 = \{2\}$. The set of participants is $N(e) = \{0,1,2,3,4\}$ and the agents' groups are $G_0(e) = \{0,1,2,3,4\}$, $G_1(e) = \{1,3\}$, $G_2(e) = \{2\}$, $G_3(e) = \{3\}$, $G_4(e) = \{4\}$, and $G_5(e) = \emptyset$. As $G = \{G_i\}_{i \in N_0}$ forms a partial ordering, it induces a tree. As none of the two referrers of agent 4 exclusively claims an contribution to the agent 4's participation, note that only the unique common predecessor can claim the contribution, that is, $4 \notin e_0$ but $4 \in F_0(e)$.

where the second equality comes from the *free exclusion* assumption, as for any v_i':

$$W(v_i', v_{-i}, N(e_i', e_{-i}')) \geq W(v_i', v_{-i}, N(e_{-i}')) = W(v, O_i(e)).$$

Under complete network information with $e_0 = N$, it is standard that

$$P_i^V(v,e) = W(v, N(e) \setminus \{i\}) - \sum_{j \in N(e) \setminus \{i\}} v_j(g(v,e)) = W(v, N \setminus \{i\}) - \sum_{j \in N \setminus \{i\}} v_j(g(v,e)).$$

In incomplete network information, however, agent i can lower $W(v, N(e) \setminus \{i\})$ by referring only part of his acquaintances $e_i' \subsetneq e_i$. Thus, the VCG payment can be decomposed in two parts:

$$P_i^V(v,e) = \Big[W(v, N(e) \setminus \{i\}) - \sum_{j \in N(e) \setminus \{i\}} v_j(g(v,e))\Big] - \Big[W(v, N(e) \setminus \{i\}) - W(v, O_i(e))\Big],$$

where the first part is the marginal effect on the others given fixed participants $N(e)$, which is equivalent to the usual VCG payment; and the second part is a reward for effective referrals. If agent i's referrals are not effective, that is $G_i(e) = \{i\}$, then the second part is always zero.

The following example illustrates how a VCG mechanism deals with incomplete network information in a simple auction setting. Remark that the VCG mechanism is different from the second-price auction which does not compensate agents for referrals.

Example 1. Consider a single-item auction with two existing buyers and one potential buyer: $N = \{1,2,3\}$, $e_0 = \{1,2\}, e_1 = \{3\}, e_2 = e_3 = \emptyset$. Thus, buyer 3 can participate

in the auction only through buyer 1's referral. Suppose the buyers' valuations to the auctioned item are x, y, and z, respectively, where the auctioneer's valuation is zero. First, consider a second-price auction in which bidding the true value is a dominant strategy, for each buyer, no matter who the participants are. Buyer 1's payoff from referring buyer 3, that is, reporting $e_1 = \{3\}$, is $\max\{x - \max\{y, z\}, 0\}$. However, his payoff from not referring, or reporting $e_1 = \emptyset$, is $\max\{x-y, 0\}$. If buyer 3 wins, buyer 1 loses the item without any compensation. Furthermore, even if buyer 1 still wins after referring buyer 2, particularly if the referred buyer 3's value z is between x and y, he must pay more due to the referral. Thus, not referring is a dominant strategy for buyer 1. In addition, the second-price auction in this setting fails to obtain an efficient allocation: buyer 1 does not refer buyer 3 and buyer 3 cannot participate in the auction although her valuation is the highest among the three buyers.
Now consider a VCG mechanism which is different from the second-price auction. If buyer 1 wins, then his payment is y, no matter whether buyer 3 participates. If buyer 3 wins, althought buyer 1 loses the item, his VCG payment is $P_1^V = W(v, \{2\}) - \{v_2(g(v, e)) + v_3(g(v, e)) = y - z$, which is negative. That means, buyer 1 will be rewarded as a compensation for referring buyer 3. Hence, referring buyer 3 is buyer 1's dominant strategy. $\square$

We confirm that a VCG mechanism is incentive compatible and individually rational in dominant strategies.

Proposition 1. *A VCG Mechanism is incentive compatible and individually rational.*

Proof. First, we show a VCG mechanism (g, P^V) is incentive compatible. Given v_{-i} and e_{-i}, consider an agent i whose private information is v_i and e_i. Reporting v_i' and $e_i' \subseteq e_i$, the agent i's payoff is

$$\begin{aligned}
v_i(g(v', e')) - P_i^V(v', e') &= v_i(g(v', e')) - \left[W(v', O_i(e')) - \sum_{j \in N(e') \setminus \{i\}} v_j(g(v', e'))\right] \\
&= \left[v_i(g(v', e')) + \sum_{j \in N(e') \setminus \{i\}} v_j(g(v', e'))\right] - W(v, O_i(e)) \\
&\leq W(v, N(e')) - W(v, O_i(e)) \\
&\leq W(v, N(e)) - W(v, O_i(e)) \\
&= v_i(g(v, e)) - P_i^V(v, e).
\end{aligned}$$

$O_i(e) = O_i(e')$ The second equality is due to and $W(v', O_i(e')) = W(v, O_i(e))$. The third line is from the fact that $v_i(g(v', e')) + \sum_{j \in N(e') \setminus \{i\}} v_j(g(v', e'))$ is maximized at $v_i' = v_i$. The last inequality is from monotonicity of W. Thus truthfully reporting both v_i and e_i is a dominant strategy for the agent i.

Next, we show (g, P^V) is individually rational. Since reporting one's private information truthfully is a dominant strategy, it suffices to show that, for any profile (v, e) and any $i \in N$,

$$v_i(g(v,e)) - P_i^V(v,e) \geq 0. \tag{1}$$

By definition of W, (1) is equivalent to $W(v, N(e)) - W(v, O_i(e)) \geq 0$. Due to *free exclusion*, $W(v, N(e)) - W(v, O_i(e))$ is nonnegative as desired. □

Remark. In a fixed population setting, it is well-known that a VCG mechanism satisfies both incentive compatibility and individual rationality. When the population is endogenously determined by the players' referrals, on the other hand, those properties rely on monotonicity of the underlying social welfare function. If an additional participant may harm the social welfare, even in a VCG mechanism, the existing agents conceal their network connections to prevent potential agents' participation and the referred agents do not want to participate because of high payments. Thus, the *free exclusion* assumption is crutial for a VCG mechanism to be individually rational and incentive compatible.

Though a VCG mechanism has desirable properties in terms of eliciting agents' private information, it may not be useful in practice due to its budget problem. In environments with a varying population, it turns out that a VCG mechanism tends to run a deficit because the designer has to compensate referrers too highly. The following proposition shows that a VCG mechanism runs a deficit if and only if the sum of the agents' marginal contributions through referrals is greater than the social welfare.

Proposition 2. *Given $(v, e) \in T \times E$, a VCG mechanism (g, P^V) runs a deficit if and only if*

$$\sum_{i \in N(e)} [W(v, N(e)) - W(v, O_i(e))] > W(v, N(e)). \tag{2}$$

Proof. We have that

$$\begin{aligned}
\sum_{i \in N} P_i^V(v,e) &= \sum_{i \in N(e)} \left[W(v, O_i(e)) - \sum_{j \in N(e)\setminus\{i\}} v_j(g(v,e)) \right] \\
&= \sum_{i \in N(e)} \left[W(v, O_i(e)) - \sum_{j \in N(e)\setminus\{i\}} v_j(g(v,e)) - v_i(g(v,e)) \right] \\
&\quad + \sum_{i \in N(e)} v_i(g(v,e)) \\
&= \sum_{i \in N(e)} [W(v, O_i(e)) - W(v, N(e))] + W(v, N(e)).
\end{aligned}$$

Thus, we have $\sum_{i \in N} P_i^V(v,e) < 0$ if and only if (2) holds. □

The following example illustrates that a VCG mechanism runs a deficit particularly when the new participant's contribution is relatively large.

Example 2. Consider a single-item auction as in Example 1. Due to Proposition 2, a VCG mechanism runs a deficit if and only if

$$\max\{x, y, z\} > y + \max\{x, y\}.$$

To be specific, suppose $x = 1$ and $y = 2$.

- Suppose $z = 3$. Buyer 3 wins the item and pays $P_3^V = 2$ which is the highest bid among $O_3(e) = \{1, 2\}$. Buyer 2 has no marginal effect on the welfare and hence pays nothing. The payment of buyer 1 is the difference between the highest bid $y = 2$ among $O_1(e) = \{2\}$ and the highest bids $z = 3$ among $N(e) \setminus \{1\}$. Thus $P_1^V = y - z = -1$. That means the auctioneer must pay 1 to buyer 1 as a compensation for the referral. The auctioneer's net revenue is $P_1^V + P_2^V + P_3^V = -1 + 0 + 2 = 1$: it runs a surplus.

- Suppose $z = 5$. Buyer 3 wins the item paying $P_3^V = 2$ and buyer 2 pays nothing as before. The payment of buyer 1 is the difference between the highest bid under his absence, which is $y = 2$ again same as before, and the highest bids among $N(e) \setminus \{1\}$, which is now $z = 5$. Thus $P_1^V = -3$ and the auctioneer must pay 3 to buyer 1 as a compensation for referral: it runs a deficit due to the negative net payment, $P_1^V + P_2^V + P_3^V = -3 + 0 + 2 = -1$. $\square$

Remark. Rearranging the terms, note that (2) is equivalent to

$$W(v, N(e)) > \sum_{i \in N(e)} \frac{W(v, O_i(e))}{|N(e)| - 1}.$$

This implies that a VCG mechanism runs a deficit if and only if the social welfare is greater than the sum of the agents' marginal contribution to the *per-capita welfare*.

The following corollary investigates the relation between the budget problem of a VCG mechanism and the existence of a core allocation of the underlying environment. According to the notion of *network-restricted games* proposed by Myerson (1977), given (v, e), we define a *network-restricted per-capita game* $(N(e), w^{v,e})$: for each $S \subset N(e)$,

$$w^{v,e}(S) = \frac{W(v, S \cap N(e_S))}{|S \cap N(e_S)|}.$$

Corollary 1. *Given (v, e), if a VCG mechanism yields a strictly positive surplus, the core of the network-restricted per-capita game $(N(e), w^{v,e})$ must be empty.*

Proof. We prove the contrapositive of the statement. Suppose that the core of $(N(e), w^{v,e})$ is nonempty. Due to Bondareva (1963) and Shapley (1967), for any probability measure δ on $2^{N(e)} \setminus \{\emptyset, N(e)\}$,

$$\sum_{S \subsetneq N(e)} \delta(S) w^{v,e}(S) \leq w^{v,e}(N(e)) = \frac{W(v, N(e))}{|N(e)|}. \tag{3}$$

Now define δ by:

$$\delta(S) = \begin{cases} \frac{1}{|N(e)|} & \text{if } S = N(e) \setminus \{i\} \text{ and } i \in N(e) \\ 0 & \text{otherwise.} \end{cases}$$

With this probability measure δ, the left hand side of (3) becomes

$$\sum_{S \subsetneq N(e)} \delta(S) w^{v,e}(S) = \sum_{i \in N(e)} \frac{1}{|N(e)|} \frac{W(v, O_i(e))}{|N(e)| - 1}$$

and it follows

$$\sum_{i \in N(e)} \frac{W(v, O_i(e))}{|N(e)| - 1} \leq W(v, N(e)). \tag{4}$$

Due to Proposition 2, (4) implies that a surplus from a VCG mechanism must be less than or equal to zero. □

The following example shows how a budget problem could be significant under VCG mechanisms.

Example 3. [Rivalry Good] A planner wants to provide a rivalry good, such as a small swimming pool. Note that a set of allocations is $\Gamma = 2^N$. All the agents, who do not like congestion, have the same preference:

$$v_i(S) = \begin{cases} 6 - |S| & \text{if } i \in S \\ 0 & \text{otherwise.} \end{cases}$$

Suppose the agents are connected in a chain network, so $e_i = \{i + 1\}$ for each $i \geq 0$. Suppose only the private information of each agent is about his connection, but not his preference. Under a VCG mechanism, agent 1 refers agent 2 and agent 2 refers agent 3, but agent 3 does not refer agent 4 and so on. Thus, three agents participate and the allocation $\gamma = \{1, 2, 3\}$ is socially optimal as $W(\{1, 2, 3\}) = 3 \times 3 = 9 > W(S)$ for any $|S| \neq 3$. Each agent's payment is:

- $P_1 = W(\emptyset) - (v_2(N) + v_3(N)) = 0 - 6 = -6$
- $P_2 = W(\{1\}) - (v_1(N) + v_3(N)) = 5 - 6 = -1$
- $P_3 = W(\{1, 2\}) - (v_1(N) + v_2(N)) = 8 - 6 = 2,$

which means agent 3 pays 2, but agent 1 and agent 2 should be rewarded by 6 and 1 from the planner as they have been positively contributed to the society through referrals. The payoff for each agent is $(3 + 6, 3 + 1, 3 - 2) = (9, 4, 1)$, however, the sum of the payments is $-6 - 1 + 2 = -5$, which means the planner must run a deficit. □

Remark. In Example 3, it is important to note that the socially optimal participants are endogenously determined under a VCG mechanism, without assuming *free exclusion.* If a planner approaches the potential agents by public advertisements instead of sequential referrals, any agent will join in as long as the current participants are less than 6. Thus the allocation with public advertisements is suboptimal due to congestion.

4 A Multilevel Mechanism

This section introduces an alternative mechanism, namely a *multilevel mechanism*, which guarantees a nonnegative surplus to the mechanism designer. As we have seen in the previous section, under a VCG mechanism the mechanism designer provides *too much* compensation to agents who contribute to new agents' participation, and hence it generically runs a deficit. Under a multilevel mechanism, an agent is compensated with an appropriate transfer not from the mechanism designer but from his referred agents, who would not be able to participate without him.

Definition 1 (Multilevel Mechanism)**.** A multilevel mechanism consists of an efficient allocation rule g and a payment rule P^M which satisfies, for all $i \in N(e)$,

$$P_i^M(v,e) = P_i^G(v,e) - \sum_{j \in F_i(e)} P_j^G(v,e),$$

where $P_i^G(v,e) = W(v, O_i(e)) - \sum_{j \in O_i(e)} v_j(g(v,e))$.

Remark. Note that $P_i^G(v,e)$ is the marginal effect of the agent i's group on the society. This is different from the VCG payment $P_i^V(v,e)$, which is the marginal effect of i alone. Under a multilevel mechanism, an agent i pays $P_i^G(v,e)$ to i's immediate predecessor. Only the agents in $F_0(e)$ pay to the mechanism designer. That is, each agent is responsible for his group when he pays to his predecessor, but he is compensated by transfers from his referred agents as they pays their group-wise payment to him.

The main benefit of such a simple multilevel mechanism is that it guarantees a nonnegative revenue to the designer. The following example illustrates how an auctioneer makes a positive surplus with a multilevel mechanism; while an auctioneer runs a deficit with a VCG mechanism.

Example 4 (VCG mechanism vs. Multilevel mechanism)**.** Consider a single-item auction with two existing buyers, A and B, and two potential buyers, C and D, who are connected to A as shown in Figure 2. We first note that under both a VCG mechanism and a multilevel mechanism, fully referring is a dominant strategy: hence, all four players are supposed to participate in the mechanisms. To be specific, suppose they bid 5, 2, 7, and 10, respectively.

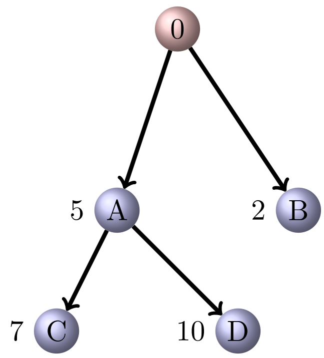


Figure 2: A 4-buyer auction (Example 4)
Suppose A refers C and D and each buyer bids 5, 2, 7, and 10, respectively. Under both a VCG mechanism and a multilevel mechanism, the item goes to D, who is the highest bidder. Under a VCG mechanism, the winner D pays 7, which is the second highest bid, to the auctioneer, but the auctioneer must compensate the referrer A by rewarding $10 - 2 = 8$, the marginal contribution of A; and hence the auctioneer runs a deficit by -1. Under a multilevel mechanism, however, the winner D pays the price 7 to the referrer A, and the referrer A pays the highest bid 2 among the outsiders of A to the auctioneer; thus the auctioneer's net revenue is 2.

- *A VCG mechanism:* Buyer D wins the item and pays 7, which is the second highest bid, to the auctioneer. However, buyer A's payment, which is the difference between the highest bid among $O_A(e) = \{B\}$ and the highest bid among $N(e) \setminus \{A\} = \{B, C, D\}$, is $2 - 10 = -8$. Thus the auctioneer must pay 8 to the buyer A as a reward and her net revenue is $7 - 8 = -1$.

- *A multilevel mechanism:* Buyer D wins the item and pays 7 to his referrer A, rather than paying to the auctioneer. Then buyer A pays 2, which is the highest bid among $O_A(e) = \{B\}$, to the auctioneer. Thus the auctioneer's net revenue is 2. □

The first main result for a multilevel mechanism is that it guarantees a nonnegative surplus to the mechanism designer.

Proposition 3. *A multilevel mechanism always runs a budget surplus.*

Proof. First remark that, for any agent i, his group-wise payment $P_i^G(v, e)$ is always nonnegative, as

$$P_i^G(v, e) = W(v, O_i(e)) - \sum_{j \in O_i(e)} v_j(g(v, e)) \geq 0.$$

Then, for any (v, e), the sum of the payments from the agents must be nonnegative, as

$$\sum_{i \in N(e)} P_i^M(v, e) = \sum_{i \in F_0(e)} P_i^G(v, e).$$

□

To further investigate incentive properties of a multilevel mechanism, Lemma 1 below shows preliminary results on an agent's payoff. For notational simplicity, define:

- for any agent i and any reported profile (v, e):

$$\Delta W_i(v', e') = \begin{bmatrix} W(v', N(e')) \\ -W(v', O_i(e')) \end{bmatrix} - \sum_{j \in F_i(e')} \begin{bmatrix} W(v', N(e')) \\ -W(v', O_j(e')) \end{bmatrix},$$

which is the difference between the *marginal contribution of i's referrals* and the *sum of the marginal contributions of i's referred agents' referrals.*

- for any agent i with the preference of v_i and any reported profile (v', e'):

$$\Delta v_i(g(v', e')) = v_i'(g(v', e')) - v_i(g(v', e')),$$

which is the difference between agent i's *hypothetical value* and his *true value* on the proposed allocation $g(v', e')$ by manipulation.

Lemma 1. *Suppose that an agent i's true type is* (v_i, e_i) *and others report* (v_{-i}, e_{-i})*. Under the multilevel mechanism,*

i) the agent i's ex-post payment from truth telling is

$$P_i^M(v, e) = W(v, O_i(e)) + \sum_{j \in F_i(e)} [W(v, N(e)) - W(v, O_j(e))] - \sum_{j \in N(e) \setminus \{i\}} v_j(g(v, e)));$$

ii) the agent i's ex-post payoff from truth telling is,

$$v_i(g(v, e)) - P_i^M(v, e) = \Delta W_i(v', e').$$

iii) the agent i's ex-post payoff from reporting $(v_i', e_i') \in T_i \times E_i$ *is*

$$v_i(g(v', e')) - P_i^M(v', e') = \Delta W_i(v', e') - \Delta v_i(g(v', e'));$$

Proof. Note that

$$\begin{aligned}
P_i^M(v, e) &= P_i^G(v, e) - \sum_{j \in F_i(e)} P_j^G(v, e) \\
&= \left(W(v, O_i(e)) - \sum_{j \in O_i(e)} v_j(g(v, e)) \right) - \sum_{j \in F_i(e)} \left(W(v, O_j(e)) - \sum_{k \in O_j(e)} v_k(g(v, e)) \right) \\
&= \left(W(v, O_i(e)) - \sum_{j \in O_i(e)} v_j(g(v, e)) \right) \\
&\quad - \sum_{j \in F_i(e)} [W(v, O_j(e)) - W(v, N(e))] - \sum_{j \in F_i(e)} \sum_{k \in G_j(e)} v_k(g(v, e)) \\
&= - \sum_{j \in N(e) \setminus \{i\}} v_j(g(v, e))) + W(v, O_i(e)) + \sum_{j \in F_i(e)} [W(v, N(e)) - W(v, O_j(e))],
\end{aligned}$$

where the third equality is due to $\sum_{k \in O_j(e)} v_k(g(v,e)) = W(v,N(e)) - \sum_{k \in G_j(e)} v_k(g(v,e))$ and the last equality is from $O_i(e) \cup \left(\cup_{j \in F_i(e)} G_j(e)\right) = N(e) \setminus \{i\}$. This completes the first part.

Using the first part, we have

$$\begin{aligned} v_i(g(v,e)) - P_i^M(v,e) &= W(v,N(e)) - W(v,O_i(e)) - \sum_{j \in F_i(e)} [W(v,N(e)) - W(v,O_j(e))] \\ &= \Delta W_i(v,e), \end{aligned}$$

which completes the second part.

When the agent i reports (v_i', e_i'), his payoff is $v_i(g(v',e')) - P_i^M(v',e')$. Subtracting and adding $v_i(g(v',e'))$, we have

$$\begin{aligned} v_i(g(v',e')) - P_i^M(v',e') &= v_i(g(v',e')) - v_i(g(v',e')) + v_i(g(v',e')) - P_i^M(v',e') \\ &= \Delta W_i(v',e') - \Delta v_i(g(v',e')), \end{aligned}$$

which completes the last part. □

Due to Lemma 1, Proposition 4 below is straightforward to show a multilevel mechanism is individually rational.

Proposition 4. *A multilevel mechanism is individually rational.*

Proof. By Lemma 1, agent i's payoff from reporting truth preference and referring $\emptyset$ is:

$$\begin{aligned} \Delta W_i(v, e_{-i}) &= \begin{bmatrix} W(v, N(e_{-i})) \\ -W(v, O_i(e_{-i})) \end{bmatrix} - \sum_{j \in F_i(e_{-i})} \begin{bmatrix} W(v, N(e_{-i})) \\ -W(v, O_j(e_{-i})) \end{bmatrix} \\ &= W(v, N(e_{-i})) - W(v, N(e_{-i}) \setminus \{i\}), \end{aligned}$$

which is nonnegative, noting that $F_i(e_{-i}) = \emptyset$. In equilibrium, therefore, a positive payoff is guaranteed regardless others' types and strategies. □

If an agent's marginal contribution increases as population grows, agents may want to spontaneously refer other potential agents to increase the population or the designer can align agents' interest in the same direction to make all the existing agents happy with new agents' participation.[2] However, we are mainly interested in environments with potential conflict of interests among the mechanism designer and the agents on varying participants. It turns out that multilevel mechanisms work well in *submodular* environments in which each individual's marginal contributions to the society get smaller as population grows.

Definition 2. A welfare function W is *submodular* if for all $S' \subseteq S$ and all $v' \leq v$:

$$W(v, S') - W(v', S') \geq W(v, S) - W(v', S)$$

[2] Sprumont (1990) shows that one can find a *population monotonic allocation scheme* in a *supermodular* environment.

Remark. A submodular welfare function requires that "a higher preference profile is worth more in a smaller coalition than in a larger coalition" as well as that "increasing population is worth more when they have a lower preference profile." Note also that supermodularity implies *submodular in coalitions*, that is, for all $i \notin S' \subseteq S$:

$$W(v, S' \cup \{i\}) - W(v, S') \geq W(v, S \cup \{i\}) - W(v, S)$$

The following lemma characterizes submodularity of a general set function, namely *Exclusion-Inclusion Principal for Submodularity.*

Lemma 2. *A set function W defined on 2^N is submodular if and only if, for all $S, S' \subseteq N$ such that $S' \subseteq S$ and all partition $\mathcal{P}$ of $S \setminus S'$,*

$$W(S) - W(S') \geq \sum_{P \in \mathcal{P}} [W(S) - W(S \setminus P)] . \tag{5}$$

Proof. First, we prove the 'only-if' part by mathematical induction. Let S be an arbitrary subset of N. If $S' = S$, then the claim is obvious. Suppose that $S \setminus S'$ is a singleton, say $S \setminus S' = \{k\}$. Then the claim (5) trivially holds. As an induction hypothesis, suppose that, for all $S' \subsetneq S$ such that $|S \setminus S'| \leq l$, the statement (5) is true. Now consider a subset $S' \subsetneq S$ such that $|S \setminus S'| = l + 1$ and a partition $\mathcal{P}$ of $S \setminus S'$. Pick any $k \in S \setminus S'$ and let $P_1 \in \mathcal{P}$ such that $k \in P_1$. Define a new partition $\tilde{\mathcal{P}}$ of $S \setminus (S' \cup \{k\})$ by replacing P_1 with $\tilde{P}_1 \equiv P_1 \setminus \{k\}$, that is, $\tilde{\mathcal{P}} = (\mathcal{P} \setminus \{P_1\}) \cup \{\tilde{P}_1\}$. Note that $\tilde{P}_1$ is possibly empty. Since $|S \setminus (S' \cup \{k\})| \leq l$, we have that

$$W(S) - W(S' \cup \{k\}) \geq \sum_{P \in \tilde{\mathcal{P}}} [W(S) - W(S \setminus P)] .$$

Subtracting $W(S') - W(S' \cup \{k\})$ from the both side, we have that

$$\begin{aligned} W(S) - W(S') &\geq \sum_{P \in \tilde{\mathcal{P}}} [W(S) - W(S \setminus P)] - [W(S') - W(S' \cup \{k\})] \\ &= \sum_{P \in \mathcal{P} \setminus \{P_1\}} [W(S) - W(S \setminus P)] \\ &\quad + W(S) - W(S \setminus \tilde{P}_1) - [W(S') - W(S' \cup \{k\})]. \end{aligned} \tag{6}$$

On the other hand, by submodularity, we have

$$W(S' \cup \{k\}) + W(S \setminus P_1) \geq W(S') + W(S \setminus \tilde{P}_1). \tag{7}$$

Plugging (7) into (6), we have

$$W(S) - W(S') \geq \sum_{P \in \mathcal{P}} [W(S) - W(S \setminus P)] ,$$

which implies that the claim (5) holds for $S' \subset S$ such that $|S \setminus S'| = l + 1$ and the given $\mathcal{P}$. Since we have chosen S' and $\mathcal{P}$ arbitrarily, we get the desired conclusion.

Next, we prove the 'if' part. Pick any $U, V \subseteq N$. Let $S' = U \cap V$, $S = U \cup V$, and $\mathcal{P} = \{U \setminus S', V \setminus S'\}$. The condition (5) implies that

$$W(S) - W(S') \geq [W(S) - W(V)] + [W(S) - W(U)],$$

or equivalently, $W(U) + W(V) \geq W(S) + W(S')$, as desired. □

One of the good properties of a multilevel mechanism is that each agent has no incentive to under-report his preference. Although a multilevel mechanism is not fully incentive compatible with truthfully reporting preference, this property is good at least for a mechanism designer in terms of her budget.

Proposition 5. *Suppose W is submodular. For all (v, e) and all $i \in N(e)$, under-reporting i's preference is dominated by reporting his true preference.*

Proof. Given (v, e) and $i \in N(e)$, due to Lemma 1, the agent i's from reporting v_i' consists of three parts:

$$\begin{aligned} v_i(g(v', e)) - P_i^M(v', e) \quad = \quad & v_i(g(v', e)) + \sum_{j \in N(e) \setminus \{i\}} v_j(g(v', e))) && (8) \\ & - W(v', O_i(e)) && (9) \\ & - \sum_{j \in F_i(e)} \left[W(v', N(e)) - W(v', O_j(e)) \right]. && (10) \end{aligned}$$

The first part (8) is maximized at $v_i' = v_i$ and the second part (9) does not depend on v_i'. Since W is submodular, for all $j \in F_i(e)$, if $v_i' < v_i$, then

$$W(v', N(e)) - W(v', O_j(e)) \geq W(v, N(e)) - W(v, O_j(e)).$$

Thus, if $v_i' < v_i$, then the last part (10) is less than or equals to

$$- \sum_{j \in F_i(e)} \left[W(v, N(e)) - W(v, O_j(e)) \right],$$

which is from truth-telling. □

Under multilevel mechanisms, under-reporting their preference is dominated, but they may over-report their preference. The next proposition characterizes an agent's belief against which reporting his true preference is his best response. It turns out agents should report their true preference if they have pessimistic beliefs about their referred agents.

Proposition 6. *For any player $i \in N$, reporting the true preference is a best response if the player i believes that, for all v_i',*

$$W(v', N(e)) = W(v', O_i(e) \cup \{i\}). \tag{11}$$

Proof. It suffices to show (10) is zero. Since $O_i(e) \cup \{i\} \subseteq O_j(e) \subseteq N(e)$ for all $j \in F_i(e)$, monotonicity of W implies that, for all $t'_i \in T_i$,

$$W(v', O_i(e) \cup \{i\}) \leq W(v', O_j(e)) \leq W(v', N(e)).$$

However, the condition (11) yields that, for all $j \in F_i(e)$,

$$W(v', O_j(e)) = W(v', N(e)),$$

as desired. □

Remark. The condition (11) is implied by some particular situations. If an agent i has no other agents to refer, it is clear that truthfully reporting is a dominant strategy. Though i refers others, if she believes that they could also be referred by some other existing agents, then i cannot claim a contribution from the new participants and $F_i(e) = \emptyset$ and the condition (11) holds. Although she could exclusively refer new participants, if they do not increase the actual social welfare then she has no incentive to tell a lie. Therefore, if agents are both *ambiguity-averse* and they also cannot exclude such possibilities then truthfully reporting their preference is a dominant strategy.

Now we study the incentive properties on revealing agents' connection. Due to Lemma 1, incentive compatibility for an agent i on his connection is: for any v' and any $e'_i \subseteq e_i$,

$$v_i(g(v', e)) - P_i^M(v', e) \geq v_i(g(v', e')) - P_i^M(v', e'),$$

or equivalently,

$$\Delta W_i(v', e) - \Delta v_i(g(v', e)) \geq \Delta W_i(v', e') - \Delta v_i(g(v', e')) \tag{12}$$

As a preliminary result, we confirm that, for each agent, fully-revealing his connection is his dominant strategy assuming he truthfully reports his preference.

Lemma 3. *Suppose W is submodular. For any v, we have*

$$\left[e'_i \subseteq e_i\right] \implies \Delta W_i(v, e) \geq \Delta W_i(v, e')$$

Proof. Recall that

$$\begin{aligned} \Delta W_i(v, e) &= \begin{bmatrix} W(v, N(e)) \\ -W(v, O_i(e)) \end{bmatrix} - \sum_{j \in F_i(e)} \begin{bmatrix} W(v, N(e)) \\ -W(v, O_j(e)) \end{bmatrix} \quad \text{and} \\ \Delta W_i(v, e') &= \begin{bmatrix} W(v, N(e')) \\ -W(v, O_i(e')) \end{bmatrix} - \sum_{j \in F_i(e')} \begin{bmatrix} W(v, N(e')) \\ -W(v, O_j(e')) \end{bmatrix}. \end{aligned}$$

Take $e'_i \subseteq e_i$. Then it follows $F_i(e') \subseteq F_i(e)$ and $O_i(e) = O_i(e')$. Thus, $\Delta W_i(v, e) \geq \Delta W_i(v, e')$ is equivalent to

$$\begin{bmatrix} W(v, N(e)) \\ -W(v, N(e')) \end{bmatrix} + \sum_{j \in F_i(e')} \begin{bmatrix} W(v, N(e')) + W(v, O_j(e)) \\ -W(v, N(e)) - W(v, O_j(e')) \end{bmatrix} \geq \sum_{j \in F_i(e) \setminus F_i(e')} \begin{bmatrix} W(v, N(e)) \\ -W(v, O_j(e)) \end{bmatrix}. \quad (13)$$

Since $N(e') \cup O_j(e) = N(e)$ and $N(e') \cap O_j(e) = O_j(e')$ for all $j \in F_i(e')$, submodularity of W implies:

$$\begin{bmatrix} W(v, N(e')) + W(v, O_j(e)) \\ -W(v, N(e)) - W(v, O_j(e')) \end{bmatrix} \geq 0. \quad (14)$$

Applying Lemma 2, submodularity of W implies:

$$W(v, N(e)) - W(v, N(e')) \geq \sum_{j \in F_i(e) \setminus F_i(e')} \begin{bmatrix} W(v, N(e)) \\ -W(v, O_j(e)) \end{bmatrix}. \quad (15)$$

(14) and (15) jointly implies (13), which completes the proof. □

Proposition 7. *Suppose W is submodular. For any agent, it is a dominant strategy to fully reveal his connection, provided that he reports his true preference.*

Proof. If agent i truthfully reports his true preference v_i, then $\Delta v_i(g(v, e)) = v_i(g(v'e)) - v_i(g(v'e)) = 0$. Thus, the incentive compatibility condition (12) for fully revealing connection is: for any $i \in N(e)$ and $e'_i \subseteq e_i$,

$$\Delta W_i(v, e) \geq \Delta W_i(v, e'). \quad (16)$$

As W is submodular, Lemma 3 implies (16). □

The following corollary is a direct consequence of Proposition 7.

Corollary 2. *Suppose W is submodular. For any agent with no private information about preference, it is a dominant strategy to fully reveal his connection.*

Example 5. [Rivalry Good Revisited] Consider the rivalry good provision problem as in Example 3. Fully revealing connection is a dominant strategy as long as it increases the social welfare. Hence, as in Example 3, agent 1 refers agent 2 and agent 2 refers agent 3. Under a multilevel mechanism, each agent's group-wise payment is as follows:

- $P_1^G = W(\emptyset) - 0 = 0 - 0 = 0$
- $P_2^G = W(\{1\}) - v_1(N) = 5 - 3 = 2$
- $P_3^G = W(\{1, 2\}) - (v_1(N) + v_2(N)) = 8 - 6 = 2$

which implies, agent 3 pays 2 to agent 2, agent 2 pays 2 to agent 1, and agent 1 pays nothing to the planner. Thus their payoff is 3+2=5 for agent 1, 3-2+2=3 for agent 2, and 3-2=1 for agent 3. Now we confirm that agent 1 and agent 2 have no incentive not to reveal their connection. Suppose agent 2 does not refer agent 3. Then agent 2's value from the allocation $\{1, 2\}$ is 4 and he pays only $W(\{1\}) - v_1(\{1, 2\}) = 1$ to agent 1. However, he does not receive any reward from agent 3 and hence the payoff from not referring agnet 3 is 4-1=3, which is the same when he refers. Therefore, not referring is not profitable for agent 2. Next, suppose agent 1 does not refer agent 2. His payoff from not referring is 5, which is indifferent from referring. Thus, not referring is not profitable for agent 1.

Corollary 3. *Suppose W is submodular. For any agent $i \in N(e)$, it is a dominant strategy to fully reveal his connection if for any v' and any $e'_i \subseteq e_i$:*

$$\Delta v_i(g(v', e)) \leq \Delta v_i(g(v', e')). \tag{M}$$

Remark. The condition (M) can be interpreted as *Manipulation Monotonicity.* Note that $\Delta v_i(g(v', e)) > 0$ only when the manipulation by reporting v'_i instead of v_i is successful. Then the condition (M) implies that a successful manipulation is harder in a larger coalition. If a manipulation is successful in a larger coalition, however, it should also work in a smaller coalition.

As a sufficient condition for the condition (M), we define *submodularity* of preference. Submodular preference requires that agents' preference is more sensitive in a smaller coalition.

Definition 3 (Submodular Preference)**.** An agent i' preference is *submodular* (in coalitions), if for any v_i, v'_i, any v_{-i}, any S, T with $i \in S \subseteq T$, and any $\gamma_S \in \Gamma^*(v', S)$, $\gamma_T \in \Gamma^*(v', T)$,

$$v'_i(\gamma_S) \geq v_i(\gamma_S) \implies v'_i(\gamma_S) - v_i(\gamma_S) \geq v'_i(\gamma_T) - v_i(\gamma_T)$$

Proposition 8. *Suppose W is submodular. For any agent with submodular preference, it is a dominant strategy to fully reveal his connection.*

Remark. If an agent's preference does not depend on the population, it automatically satisfies submodularity. For instance in auctions without externalities, as an agent's valuation to the auctioned item does not depend on the other participants, referring all the other conneted agents is a dominant strategy under a multilevel mechanism.

5 Concluding Remarks

In this paper, we considered agents' asymmetric information about others' connections in mechanism design problems. Allowing asymmetric information about networks, each agent has an incentive not only to misreport his preference but also to conceal his connection, and hence the participants are endogenously determined. In such generalized environments, a VCG mechanism has good properties in terms of agents' incentive problems, but it runs a deficit. Alternatively, a multilevel mechanism has been proposed and it guarantees a nonnegative surplus.

The multilevel mechanism proposed in this paper is of course not the only one which yields a nonnegative surplus to the mechanism designer. We can define a class of mechanisms by *generalizing the compensation rule.* A multilevel mechanism admits an agent's contribution to increasing population only if it is *exclusive*; that is, if a new agent has been referred by two or more existing agents, then none of them has a right to claim the contribution of the new agent's participation. Many other alternative rules are possible. For instance, all agents who are related to the newcomer can split the contribution according to a certain portion. These generalized mechanisms give agents more incentive to refer other potential agents and still guarantee nonnegative payoffs.

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