Optimal guidelines in a delegated search model with no monetary transfers

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Abstract

In this paper we study a delegated search model in a principal agent framework without monetary transfers. The agent is delegated to buy an object that can have either low or high quality and high quality objects are costlier. The agent knows which quality is needed while the principal only knows the price distributions for each quality, and cannot observe the quality of the object purchased by the agent. Since the principal pays the price of the object, the agent is only interested in minimizing the search cost that she sustains. The principal can only decide which search rule to adopt, without having the possibility to use contingent monetary transfers to incentivize the agent to search optimally. We characterize the optimal search rule within a class in which the principal may impose a minimum number of searches to the agent. We find out that under some conditions, it is optimal for the principal to offer a menu of incentive compatible rules in which the low type agent is offered her fist best stopping rule, however, the high type agent is offered to perform a minimum number of searches in addition to his first best stopping rule.

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1 Introduction

A wide spread business practice is to ask a minimum number of quotations for the non-contract purchase of goods and services in organizations. For instance, the Procurement & Supplier Diversity Services of the University of Virginia has to solicit a minimum of six quotations for any purchase of 50,000 $ or above, and a minimum of three for purchases from 5,000$ to 50,000$.[1] UCL Internal Procurement Regulations requires, for any purchase in between £50,001 and £164,176, "a minimum of three competitive tender bids, and where practical more than three, in order to obtain the most competitive price"; these are common practices in procurement for many public organizations and governmental bodies. In a seemingly different context, the Italian law mandates a public competition for hiring lecturers and senior lecturers in public universities: a committee (composed by a majority of academics affiliated to other universities that the one that offers the position, to avoid favouritism toward internal candidates) evaluates all candidates by means of a two-step procedure: a preliminary evaluation of the curricula of all candidates, and an interview and detailed evaluation of each publication and qualification, of those who are short-listed. The law mandates that a minimum number of candidates should be short-listed, namely at least 10% or a minimum number of six (or all candidates, if they are less than six)[2].

These examples have three main features in common: first, the search for the best available alternative is delegated to an agent (or a group of agents) who differs from the principal; second, a minimum number of searches is imposed to the agent. Third, there is no money transfer from the principal to the agent. A common interpretation of the rationale for the requirement of "minimum number of searches", is that it aims to fight corruption or favouritism imposing a minimum level of competition and transparency in the selection process. However, to impose a minimum number of quotations or a minimum number of evaluations of a subset of candidates, can hardly be considered an effective way to avoid favouritism to take place. For instance, to favor a specific seller in a procure-

[1] http://www.procurement.virginia.edu/pageguidelinesforcomp
[2] Art.24 Italian law 240/2010.

ment, the buyer can look for alternative quotations that are more expensive (or with lower quality), and a hiring committee can shortlist only candidates who are worst than the one whom it aims to favor.

Our paper offers a different and novel explanation to rationalize the constraint of a ”minimum number of searches”, by means of a model of delegated search with moral hazard and adverse selection. In our model the agent and the principal have correlated preferences, but the agent cares more than the principal about the search cost, or, equivalently, does not fully internalize the gain of selecting the best alternative. The principal can impose a search rule, but cannot use monetary transfers to incentivize or punish the agent. This model perfectly fits the examples mentioned above and in general well describes the problem of procurement and hiring in public organizations. We focus here on procurement problems as the main application of the model, but we should keep in mind that the same arguments apply to the case of hiring, and in other contexts too. An agent needs an instrument (a good) to perform a task on behalf of a principal. There are two types of agents, $High$ and Low, who differ in the quality of the good, high quality and low quality, respectively, they need. Goods of different qualities have different price distributions, with high type goods being more expensive than low type goods. While high type agents need a high quality good to successfully perform their task, low quality agents' performance does not depend on the quality of the good they use. Principal and agent have aligned interests except as concerning the price of the good, which is fully paid by the principal. Since the agent bears a constant (non-monetary) search cost for each quotation she asks for but she does not pay the price, she always prefers to stop searching as soon as possible.

In the symmetric information case, in which the principal observes agent's type, and knows the price distribution for each quality, she can impose to each type to buy the quality she really needs and to follow the optimal (for the principal) search rule. If the search cost is constant, the optimal rule is a stopping rule with a threshold: the agent can stop searching when she finds a price lower than a given threshold. Under the assumption that high quality goods are on average more expensive than low quality goods, the optimal threshold for high type agents is higher than the optimal threshold for low type agents.

In the asymmetric information case, when the principal does neither observe the agent's type nor the quality of the purchased good, a low type agent could find it profitable to report to be a high type agent, to be allowed to buy an expensive low quality good so that to save on her search cost. It follows that the first best rule that assigns a different threshold to each type is not incentive compatible. To solve this problem, we consider a class of search rules that consists of a minimum number of searches and a threshold. The possibility of a minimum number of searches helps the principal to restore the incentive compatibility at the cost of inducing an overprovision of effort to the agents. We characterize the optimal search rule within this class. We show that under some conditions it is optimal for the principal to propose a menu of incentive compatible search rules that imposes to a high type agent a minimum number of searches. Namely, a low type agent can stop searching when she finds a price lower than her first best threshold, while the high type agent can stop search if she finds a price lower than her first best threshold, *and* she has performed a minimum number of searches.

There is a growing literature on delegated search to which this paper offers a contribution. In Armstrong and Vickers (2010) [2], the principal delegates an agent to select a project and can only decides the characteristics of the admissible projects. Mauring (2016) [9] instead, focuses on the agent's optimal policy in the case the principal cannot affect the search process directly. In her model the agent selects a set of alternatives and the principal picks her preferred one, so she analyzes the optimal stopping rule for the agent, knowing which final alternative the principal will pick from the set she proposes. Kováč et al. (2013) [7] study optimal stopping rules when the principal lacks relevant information but can consult with a better informed agent, while the exchange of contingent monetary transfers is infeasible; principal's utility from stopping depends on the state that can take only two values, either H or L: conflict of interests, as in our paper, arises because the principal prefers to stop only in state H, while the agent has always interest in stopping. In their setting, a stopping rule is such that the principal commits to a deadline. Until the deadline, the agent can make at most one proposal to stop. If the agent makes a proposal, it is accepted with some probability.

Other papers (Postl (2004) [10], Lewis (2012)[8], and Ulbricht (2016) [11]), assume transferable utility and focus on the principal's optimal contracts. Ulbricth (2016) [11] in particular shows that when the distribution of search revenues is unknown to the principal, and the search process is unobservable, search is almost surely inefficient (it is stopped too early) and second best remuneration is shown to optimally utilize a menu of simple bonus contracts.

Finally, another recent and related strand of literature (Albrecht et al.(2010) [1], Bergemann andVälimäki (2011) [3], Compte and Jehiel (2010) [4], Guler et al (2012) [5] among others), study non-homogeneous search committees that jointly decide over the continuation of a search process.

2 Model

We study a principal-agent model in which the agent needs an object of a certain quality paid fully by the principal. The principal delegates to the agent the search for the object. There are two types of agents, high type (H-agent) and low type (L-agent), depending on the quality of the object they need to carry on their task, high (H) quality and low (L) quality object, respectively. We model the situation as follows: There are two different groups of sellers to whom the agent may ask for a quote. Each seller sells an object at a given price. High (low) quality sellers sell high (low) quality objects; for simplicity of exposition and generality, we refer to the two groups of sellers as two different boxes from which the agent draws prices. Each box contains an infinite number of objects of the same quality, and, abusing notation, we denote H-box and L-box, the two boxes containing high quality and low quality objects, respectively. Let $F^a(p)$ denote the probability distribution of prices in box $a \in \{H, L\}$, which is common knowledge. For an H-type agent, objects in the H-box have a positive value $\bar{V}$, while objects in the L-box have a value equal to zero. For an L-type agent all objects, those in the H-box and those in the L-box have a positive value equal to V, where $\bar{V} \geq V$.

Let c be the cost of a search (a draw from the box), and k be the number of searches. An agent of type $a \in \{H, L\}$ has the utility

$$U^a = V^a(\hat{a}) - ck,$$

where $V^a(\hat{a})$ denote the value of an object of type $\hat{a}$ for an agent of type a, and therefore, by assumption, $V^H(L) = 0$, $V^H(H) = \bar{V}$ and $V^L(L) = V^L(H) = V$ with $\bar{V} \geq V > 0$.

It is immediate to notice that the agent does not pay the price so she aims to minimize the number of searches[3]. The principal's utility when agent is type a is

$$U^P_a = V^a(\hat{a}) - ck - p,$$

where p is the price of the object, which is entirely paid by the principal. Notice that the principal is "benevolent" because she internalizes the agent's utility, but, differently than the agent, also cares about the price paid to buy the object.

We assume that the principal cannot offer any contigent monetary transfer to the agent, and she can only propose a search rule to the agent.

The timing of the model is as follows: The principal announces the search rule for both types. Then, the agent either refuses to search and gets a payoff equal to zero, or accepts to search and reports her type to the principal. At each time $t \in \{1, 2, ...\}$, the agent draws a price p_t from a box and reports it to the principal, who decides whether the agent should stop (s) or continue (c) the search based on the search rule for that type. Formally, a search rule for type a is $R^a_t : \mathbb{R}_+ \to \{s, c\}$ for each $p_t \in \mathbb{R}_+$. By assumption, only high quality objects are valuable for an H-type agent, and therefore an H-type agent needs to search in the H-box. We assume that $F^H(p)$ first order stochastically dominates $F^L(p)$, therefore, the principal prefers that an L-type agent searches in the L-box, given that both types of objects are equally valuable for this type of agent, but objects in the L-box are in expectation cheaper.

First, we briefly analyze the standard case in which the principal knows agent's type and can observe in which box the agent searches.

[3]The model can be easily generalized to the case in which the agent pays a fraction $\lambda < \frac{1}{2}$ of the price and the principal the remaining fraction $1 - \lambda$.

3 Symmetric Information

Suppose that the principal can observe agent's type and the box from which the agent draws prices. The optimal stopping rule is such that the principal imposes to each type a to search in box a, and to stop the search when the sampled price is lower than a given threshold, which is constant over time. Hence, the optimal stopping rule for type a fixes a threshold y_a^* such that

$$\int_0^{y_a^*} (y_a^* - p) dF^a(p) = c. \tag{1}$$

The optimal mechanism with symmetric information imposes on each agent a to draw prices from box a and stop searching at time t if and only if agent a finds a price lower or equal than y_a^*. We call this search rule *a stopping rule with a threshold*.

Proposition 3.1 *If F^H first order stochastically dominates F^L, then $y_H^* \geq y_L^*$.*

Proof. Let $\phi^a(y) \equiv \int_0^y (y-p)dF^a(p)$, for $a \in \{H, L\}$. First we show that for every $y > 0$, $\phi^L(y) \geq \phi^H(y)$. Using integration by parts we have

$$\phi^a(y) = (y-p)F^a(p)\Big|_0^y + \int_0^y F^a(p)\, dp = \int_0^y F^a(p)\, dp$$

Since F^H first order stochastically dominates F^L, for every $y > 0$ we have

$$\int_0^y F^H(p)\, dp \leq \int_0^y F^L(p)\, dp$$

Therefore,

$$\phi^L(y) \geq \phi^H(y).$$

We know for y_a^* we have $\phi^a(y_a^*) = c$ for $a \in \{L, H\}$. As $\phi^L(y) \geq \phi^H(y)$ for every $y > 0$, we can conclude that $y_H^* \geq y_L^*$. ■

Let $\mathbb{E}^a(n|y)$ denote the expected number of searches in box $a \in \{L, H\}$ given a threshold y.

Proposition 3.2 *Given a threshold y, we have $\mathbb{E}^a(n|y) = \dfrac{1}{F^a(y)}$.*

Proof. We know

$$\begin{aligned}\mathbb{E}^a(n|y) &= F^a(y)+2(1-F^a(y))F^a(y)+3(1-F^a(y))^2F^a(y)+4(1-F^a(y))F^a(y)+\dots \\ &= F^a(y)\left(1+2(1-F^a(y))+3(1-F^a(y))^2+4(1-F^a(y))^3+\dots\right) \\ &= F^a(y)\left(\frac{1}{F^a(y)^2}\right)=\frac{1}{F^a(y)}\end{aligned}$$

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One can easily conclude that, if F^H first order stochastically dominates F^L, given a threshold y, we have $\mathbb{E}^L(n|y) \leq \mathbb{E}^H(n|y)$.

4 Asymmetric Information

We look now at the more interesting case in which the principal cannot observe agent's type, or monitor agent's behavior. Specifically, we have three assumptions in this Section.

Assumption 1: Principal does not observe agent's type.
This assumption creates an *adverse selection* problem.

Assumption 2: Agent can search in any of the two boxes and can at any time change the box where she is searching, and the principal cannot observe the box from which the price is drawn.

Under asymmetric information, the principal cannot propose the first best menu of stopping rules with threshold, with a different threshold for each type of agents, because this menu violates incentive compatibility. To fix the incentive compatibility problem, we give the possibility of imposing a minimum number of searches to agents. We consider a class of rules that consists of a minimum number of searches and a threshold for each type of agents. Let

$$\mathcal{R} = \{(k_L, y_L), (k_H, y_H) \mid k_a \geq 0, y_a \geq 0 \text{ for } a \in \{L, H\}\} \tag{2}$$

be the class of such rules. In other words, the agent $a \in \{L, H\}$ is asked to do at least k_a searches and if he could find a price below the threshold y_a in the first k_a searches, then the minimum of those will be used to make the purchase. Otherwise, she has to continue the search and find a price below the threshold

y_a. Also, denote $R_L = (k_L, y_L)$ and $R_H = (k_H, y_H)$. Moreover, we assume that $k_a \in \mathbb{R}_+$. When k_a is a non-integer number, one can use a random mechanism that imposes $\lfloor k_a \rfloor$, that is the floor of k_a, searches with probability one, and an additional search with probability $k_a - \lfloor k_a \rfloor$, which does not depend on the outcome of the previous searches.

We characterize the optimal rule within this class that satisfies the incentive compatible and individual rationality conditions. We do so by looking at two cases: (1) $y_L \neq y_H$, when the two types of agents are offered different thresholds: we find the optimal menu of incentive compatible search rules. (2) $y_L = y_H$, when both types of agents are offered the same thresholds. In this case, there is no need to impose a minimum number of searches to agents, because offering a common threshold $y_L = y_H$ does not cause any incentive compatibility problem. Therefore, in this case, clearly $k_L = k_H = 0$. It is enough to find the optimal common threshold. Basically, first we find the optimal rules in these two cases separately, then we provide conditions under which the menu of incentive compatible rules outperforms the common threshold rule.

We point out again that in this paper we constrain ourselves to not allow any money transfer from the principal to the agent.

Now, we discuss each of these cases in detail.

4.1 A menu of incentive compatible search rules

In this section we consider the class of rules where $y_L \neq y_H$, denoted by $\tilde{\mathcal{R}}$. Throughout the paper, by y_L^* and y_H^* we mean the first best thresholds for L-type and H-type agents respectively. Also, as in section 3 the expected number of searches in box $a \in \{L, H\}$ under threshold y is denoted by $\mathbb{E}^a(n|y)$. First, we prove the following lemma.

Lemma 4.1 *Suppose there is only one type a agent and the principal offers a search rule $R_a = (k_a, y)$ with $k_a > 0$. Then the threshold that maximizes principal's utility is y_a^*.*

Proof. Suppose the agent has already searched k_a times. Suppose the minimum price among them is higher than y_a^*: the principal can buy the good at that price or ask the agent to search to find a price lower than y_a^*. Clearly, the principal's expected utility is larger in the latter case. Suppose now that there exists a price $\tilde{p} \leq y_a^*$. The principal's payoff if she buys the good at price $\tilde{p}$ is higher than the expected payoff if the agent continues to search. Therefore the optimal threshold for y is equal to y_a^*. ■

The next theorem shows that if in the first best it is costly enough for the H-type agent to announce to be L-type (and still searching in the H box), then one can find the optimal search rule within the class $\tilde{\mathcal{R}}$.

Theorem 4.2 *Suppose* $\mathbb{E}^L(n|y_L^*) + \mathbb{E}^H(n|y_H^*) \leq \mathbb{E}^H(n|y_L^*)$. *Then, there is a value* k_H^* *that makes the search rule* $R^* = \{(0, y_L^*), (k_H^*, y_H^*)\}$ *optimal for the principal in the class* $\tilde{\mathcal{R}}$.

Proof. Let $\rho \in (0,1)$ be the probability that the agent is L-type and $1-\rho$ be the probability that the agent is H-type. We would like to find $\{(k_L, y_L), (k_H, y_H)\}$ that maximizes the principal's expected utility which is

$$\rho\left(V - c\mathbb{E}^L(n|R_L) - \mathbb{E}^L(p|R_L)\right) + (1-\rho)\left(\bar{V} - c\mathbb{E}^H(n|R_H) - \mathbb{E}^H(p|R_H)\right)$$

subject to IC conditions:

1. $\mathbb{E}^L(n|R_L) \leq \mathbb{E}^H(n|R_L)$,
2. $\mathbb{E}^L(n|R_L) \leq \mathbb{E}^L(n|R_H)$,
3. $\mathbb{E}^L(n|R_L) \leq \mathbb{E}^H(n|R_H)$,
4. $\mathbb{E}^H(n|R_H) \leq \mathbb{E}^H(n|R_L)$,

It is easy to see that the principal's expected utility without the constraints above is maximized at $\{(0, y_L^*), (0, y_H^*)\}$. Therefore, first we find the minimum values for k_L and k_H that satisfy the constraints above. One can derive

$$\mathbb{E}^a(n|R_b) = k_b + \frac{(1 - F^a(y_b))^{k_b}}{F^a(y_b)},$$

in which $a, b \in \{L, H\}$. Condition (1) holds, because we assume that F^H stochastically dominates F^L. Also, due to this assumption, it is easy to see that condition (2) implies condition (3). Therefore, for IC conditions it is enough to have (2) and (4). From (2) we have

$$k_L + \frac{(1-F^L(y_L))^{k_L}}{F^L(y_L)} \leq k_H + \frac{(1-F^L(y_H))^{k_H}}{F^L(y_H)},$$

which implies

$$k_H - k_L \geq \frac{(1-F^L(y_L))^{k_L}}{F^L(y_L)} - \frac{(1-F^L(y_H))^{k_H}}{F^L(y_H)},$$

Similarly, from (4) we have

$$k_H + \frac{(1-F^H(y_H))^{k_H}}{F^H(y_H)} \leq k_L + \frac{(1-F^H(y_L))^{k_L}}{F^H(y_L)},$$

and implies

$$k_H - k_L \leq \frac{(1-F^H(y_L))^{k_L}}{F^H(y_L)} - \frac{(1-F^H(y_H))^{k_H}}{F^H(y_H)}.$$

Therefore, the IC condition is:

$$\frac{(1-F^L(y_L))^{k_L}}{F^L(y_L)} - \frac{(1-F^L(y_H))^{k_H}}{F^L(y_H)} \leq k_H - k_L \leq \frac{(1-F^H(y_L))^{k_L}}{F^H(y_L)} - \frac{(1-F^H(y_H))^{k_H}}{F^H(y_H)}.$$

Now we need to find the minimum values for k_H and k_L that satisfies the above inequalities. Clearly, the best is to choose $k_L = 0$. Having this, the IC condition is

$$\frac{1}{F^L(y_L)} - \frac{(1-F^L(y_H))^{k_H}}{F^L(y_H)} \leq k_H \leq \frac{1}{F^H(y_L)} - \frac{(1-F^H(y_H))^{k_H}}{F^H(y_H)}. \quad (3)$$

The minimum value for k_H comes form solving the equation below:

$$\frac{1}{F^L(y_L)} - \frac{(1-F^L(y_H))^{k_H}}{F^L(y_H)} = k_H.$$

As $k_L^* = 0$, the optimal threshold y_L is y_L^*, i.e. the first best threshold for L-type agent. Moreover, by Lemma 4.1 we conclude that the optimal threshold for y_H is y_H^*. Therefore, the minimum value for k_H is denoted by k_H^* and solves

$$\frac{1}{F^L(y_L^*)} = k_H^* + \frac{(1-F^L(y_H^*))^{k_H^*}}{F^L(y_H^*)}.$$

If we write $R_H^* = (k_H^*, y_H^*)$, then k_H^* solves

$$\mathbb{E}^L(n|y_L^*) = \mathbb{E}^L(n|R_H^*).$$

Now, we show that if $\mathbb{E}^L(n|y_L^*) + \mathbb{E}^H(n|y_H^*) \leq \mathbb{E}^H(n|y_L^*)$, then the existence of the optimal value k_H^* is guaranteed. To this end, it is enough to show

$$\frac{1}{F^L(y_L^*)} - \frac{(1-F^L(y_H^*))^{k_H^*}}{F^L(y_H^*)} \leq \frac{1}{F^H(y_L^*)} - \frac{(1-F^H(y_H^*))^{k_H^*}}{F^H(y_H^*)},$$

which comes from IC condition (3). We have

$$\begin{aligned}
& \mathbb{E}^L(n|y_L^*) + \mathbb{E}^H(n|y_H^*) \leq \mathbb{E}^H(n|y_L^*) \\
\Rightarrow \quad & \frac{1}{F^L(y_L^*)} + \frac{1}{F^H(y_H^*)} \leq \frac{1}{F^H(y_L^*)} \\
\Rightarrow \quad & \frac{1}{F^L(y_L^*)} \leq \frac{1}{F^H(y_L^*)} - \frac{1}{F^H(y_H^*)} \\
\Rightarrow \quad & \frac{1}{F^L(y_L^*)} - \frac{(1-F^L(y_H^*))^{k_H^*}}{F^L(y_H^*)} \leq \frac{1}{F^L(y_L^*)} \\
\leq \quad & \frac{1}{F^H(y_L^*)} - \frac{1}{F^H(y_H^*)} \leq \frac{1}{F^H(y_L^*)} - \frac{(1-F^H(y_H^*))^{k_H^*}}{F^H(y_H^*)}
\end{aligned}$$

Therefore,

$$\frac{1}{F^L(y_L^*)} - \frac{(1-F^L(y_H^*))^{k_H^*}}{F^L(y_H^*)} \leq \frac{1}{F^H(y_L^*)} - \frac{(1-F^H(y_H^*))^{k_H^*}}{F^H(y_H^*)}.$$

The proof is complete. ■

4.2 Common threshold

In this section we analyze the best common threshold that the principal can impose if she proposes a stopping rule with a threshold to both types of agents.

Abusing notation let $\mathbb{E}U^P(y)$ denote principal's expected utility when she proposes a stopping rule with a common threshold y. Note that the principal's expected utilities from the L-type agent and the H-type agent are maximized at y_L^* and y_H^* respectively. To avoid non-interesting cases, we assume that $y_L^* < y_H^*$, and that $U_a^P(y_a^*) > 0$ for both $a \in \{L, H\}$; the first assumption implies the menu of first best stopping rules is not IC; the second assumption guarantees that when there is no problem of asymmetric information, the principal is interested that both types of agents accept to search. Let $\rho \in (0,1)$ be the probability that the agent is L-type and $1-\rho$ be the probability that the agent is H-type. Principal's expected utility is

$$\begin{aligned}\mathbb{E}U^P(y) &= \rho\left(V - c\mathbb{E}^L(n|y) - \mathbb{E}^L(p|y)\right) + (1-\rho)\left(\bar{V} - c\mathbb{E}^H(n|y) - \mathbb{E}^H(p|y)\right) \\ &= \rho U_L^P(y) + (1-\rho)U_H^P(y),\end{aligned}$$

where U_L^P and U_H^P are principal's expected utilities from the L-type agent and the H-type agent, respectively. Also, let y_H^m be the smallest threshold such that the H-type agent participates (if a smaller threshold is proposed to an H-type agent, his expected cost of search are higher than $\bar{V}$). By assumption $U_a^P(y_a^*) > 0$ for both $a \in \{L, H\}$, so it follows that $y_H^m < y_H^*$. Two cases must be considered:

First, suppose that $y_L^* < y_H^m$. In this case, if the principal proposes to both types of agents a stopping rule with threshold equal to y_L^*, an H-type agent would prefer to refuse to search. The principal then has to choose between imposing the threshold y_L^* excluding H-type agents, or a threshold that allows H-type agents' participation, which of course, will induce L-type agent to search less than optimally.

Proposition 4.3 *Assume $y_L^* < y_H^m$. If*

$$(1-\rho)U_H^P(y^*) < \rho\left(U_L^P(y_L^*) - U_H^P(y^*)\right),$$

then the principal weakly prefers to propose a stopping rule with threshold y_L^* such that only L-type agent accepts to search, to a stopping rule with a threshold $y^* \in [y_H^m, y_H^*)$ such that both types of agents accept to search. If

$$(1-\rho)U_H^P(y^*) \geq \rho\left(U_L^P(y_L^*) - U_H^P(y^*)\right),$$

then the principal weakly prefers to propose the latter stopping rule to the former.

Proof. We want to maximize $\mathbb{E}U^P(y)$ on $[0,\infty)$. We know that $U_a^P(y)$ increases for $y < y_a^*$ and decreases for $y > y_a^*$ where $a \in \{L, H\}$. Also, we know that $\mathbb{E}U^P(y)$ is differentiable on $[0,\infty)$. To maximize $\mathbb{E}U^P(y)$ we find all the points in which the first derivative of this function is zero. We call such pints critical points. Then, the maximum point would be the point that has the largest value. Therefore, we have

$$\begin{aligned}\frac{d\mathbb{E}U^P(y)}{dy} = 0 \quad &\Rightarrow \quad \rho\frac{dU_L^P(y)}{dy} + (1-\rho)\frac{dU_H^P(y)}{dy} = 0 \\ &\Rightarrow \quad \rho\frac{dU_L^P(y)}{dy} = -(1-\rho)\frac{dU_H^P(y)}{dy} \end{aligned} \tag{4}$$

As we assume $y_L^* < y_H^m$, there will be two critical points: $y = y_L^*$ and $y = y^* \in (y_L^*, y_H^*)$. Basically, y^* is the threshold that maximizes $\mathbb{E}U^P(y)$ over interval (y_L^*, y_H^*) and it is the optimal common threshold for the stopping rule that both types of agents participate. Also, we have $y^* \in [y_H^m, y_H^*)$, because for any $y < y_H^m$, the H-type agent does not participate so the choice will only distort the threshold for the L-type. Therefore, any $y \in (y_L^*, y_H^m)$ is dominated by y_L^*.

Now, if $\mathbb{E}U^P(y^*) < \mathbb{E}U^P(y_L^*)$, then y_L^* is the optimal common threshold for the stopping rule and this is true when

$$\begin{aligned}\mathbb{E}U^P(y^*) < \mathbb{E}U^P(y_L^*) \quad &\iff \quad \rho U_L^P(y^*) + (1-\rho)U_H^P(y^*) < \rho U_L^P(y_L^*) \\ &\iff \quad (1-\rho)U_H^P(y^*) < \rho\left(U_L^P(y_L^*) - U_L^P(y^*)\right)\end{aligned}$$

The proof is complete. ■

It is clear that the condition on Proposition 4.3 holds when ρ is large enough. Therefore, one can say that if the agent is of L-type with sufficiently high probability, then the principal prefers the stopping rule with the threshold y_L^*, where H-type agent does not participate, to the threshold $y^* \in [y_H^m, y_H^*)$, where both types of agents participate.

Proposition 4.4 *Assume $y_L^* \geq y_H^m$. If the principal proposes a stopping rule with a common threshold for both types, then the optimal common threshold is $y^* \in (y_L^*, y_H^*)$.*

Proof. We want to maximize $\mathbb{E}U^P(y)$ on $[0,\infty)$. We know that $U_a^P(y)$ increases for $y < y_a^*$ and decreases for $y > y_a^*$ where $a \in \{L, H\}$. Also, we know that $\mathbb{E}U^P(y)$ is differentiable on $[0,\infty)$. To maximize $\mathbb{E}U^P(y)$ we find all the points in which the first derivative of this function is zero. We call them critical points. Then, the maximum point would be the point that has the largest value. Therefore, we have

$$\begin{aligned}\frac{d\mathbb{E}U^P(y)}{dy} = 0 \quad &\Rightarrow \quad \rho\frac{dU_L^P(y)}{dy} + (1-\rho)\frac{dU_H^P(y)}{dy} = 0 \\ &\Rightarrow \quad \rho\frac{dU_L^P(y)}{dy} = -(1-\rho)\frac{dU_H^P(y)}{dy}\end{aligned}$$

As we assume $y_L^* \geq y_H^m$, there is only one critical point. Since in this case $\frac{dU_L^P}{dy}(y_H^*) \neq 0$ and $\frac{dU_H^P}{dy}(y_L^*) \neq 0$, we conclude $\mathbb{E}U_P(y)$ is maximized when $U_L^P(y)$ and $U_H^P(y)$ have opposite slopes, that is, when $U_L^P(y)$ is strictly decreasing and $U_H^P(y)$ is strictly increasing. Therefore, in this case $\mathbb{E}U^P(y)$ is maximized at some y^* such that $y_L^* < y^* < y_H^*$. ∎

The above proposition states that if given the threshold y_L^* the H-type agent participates, then the optimal common threshold for the stopping rule is a threshold strictly larger than y_L^* and strictly smaller than y_H^*.

4.3 The menu of IC rules vs. common threshold

In this section we investigate under which conditions the principal prefers to propose the optimal menu of IC rules to the optimal stopping rule with a common threshold.

Suppose first, that $y_L^* < y_H^m$ and principal is better off by imposing y_L^* than imposing $y^* \in [y_H^m, y_H^*)$. These two conditions imply that if the principal proposes a stopping rule with a common threshold, the threshold is y_L^*, and H-type agents refuse to search under this rule

It follows immediately that if principal's expected utility from having an H-type agent who follows the rule $R_H^* = (k_H^*, y_H^*)$ is non-negative, then offering the menu $R^* = \{(0, y_L^*), (k_H^*, y_H^*)\}$ is weakly better than offering a stopping rule with common threshold y_L^*. Let $\mathbb{E}U_H^P(R_H^*)$ denote principal's expected utility when she proposes the search rule R_H^* to the H-type agent.

The following proposition, that does not need a proof, summarizes this discussion.

Proposition 4.5 *Suppose* $(1-\rho)U_H^P(y^*) < \rho\left(U_L^P(y_L^*) - U_L^P(y^*)\right)$ *and* $y_L^* < y_H^m$. *If* $\mathbb{E}U^P(R_H^*) > 0$ *then principal gets higher payoff offering the menu of IC rules* R^* *than proposing the stopping rule with common threshold* y_L^*.

We now present some sufficient conditions that guarantees that H-type agent participation is beneficial for the principal when she proposes the menu of IC rules R^*. Let $x = F^H(y_H^*)$ and $u = \bar{V} - \mathbb{E}^H(p|y_H^*) - c(\mathbb{E}^H(n|y_H^*))$. Also, define $f(z) = z + \dfrac{(1-x)^z}{x} - \dfrac{1}{x}$. Now, we have the following result, which intuitively says that k_H^* should not be too large to make the rule R^* beneficial for the principal.

Proposition 4.6 *If* $k_H^* \leq f^{-1}(\frac{u}{c})$, *then we have* $\mathbb{E}U_H^P(R_H^*) \geq 0$.

Proof. Suppose $k_H^* \leq f^{-1}(\frac{u}{c})$, then we show that

$$\bar{V} - \mathbb{E}^H(p|R_H^*) - c(\mathbb{E}^H(n|R_H^*)) \geq 0.$$

Under the rule R_H^* when compared to the first best case for the H-type agent, the expected price is lower. Therefore, it is enough to show that

$$\bar{V} - \mathbb{E}^H(p|y_H^*) \geq c(\mathbb{E}^H(n|R_H^*)),$$

or equivalently to show

$$\bar{V} - \mathbb{E}^H(p|y_H^*) - c(\mathbb{E}^H(n|y_H^*)) \geq c(\mathbb{E}^H(n|R_H^*) - \mathbb{E}^H(n|y_H^*)),$$

which is the same as showing

$$\frac{u}{c} \geq \mathbb{E}^H(n|R_H^*) - \mathbb{E}^H(n|y_H^*).$$

We know that $f(z) = z + \dfrac{(1-x)^z}{x} - \dfrac{1}{x}$. Therefore, we have

$$\mathbb{E}^H(n|R_H^*) - \mathbb{E}^H(n|y_H^*) = k_H^* + \frac{(1-x)^{k_H^*}}{x} - \frac{1}{x} = f(k_H^*).$$

One can easily see that $f'(z) > 0$, for every $z \in \mathbb{R}$. Therefore, $f(z)$ is a continuous and strictly increasing function. Also, $f(0) = 0$. Then, there is a $\hat{z} > 0$ such that $f(\hat{z}) = \frac{u}{c}$. Hence, for every $k \leq \hat{z}$ we have $f(k) \leq \frac{u}{c}$. In other words, if $k_H^* \leq \hat{z}$, then $f(k_H^*) \leq \frac{u}{c}$. The proof is complete. ∎

The following example illustrates a situation in which the above sufficient condition holds. For simplicity of the calculations in the examples of this section, we consider the rule $\{(0, y_L^*), (k, y_H^*)\}$ in which $k = \lceil k_H^* \rceil$. This will serve the purpose of the section, where we provide conditions under which the menu of IC search rules outperforms the common threshold rule. In other words, if the rule $\{(0, y_L^*), (k, y_H^*)\}$ is preferred to the common threshold y^* by the principal, then the rule $\{(0, y_L^*), (k_H^*, y_H^*)\}$ will be too.

Example 4.7 Let $L = [0, 1]$ and $H = [0, 8]$ be the two boxes, with uniform price distributions on both intervals: $F^L(y) = y$ and $F^H(y) = \frac{y}{8}$. Assume $V = \bar{V} = 2$, $c = 0.2$, and $\rho = 0.7$. From equation (1), one can derive $y_L^* = 0.63$ and $y_H^* = 1.79$. Also, one can easily find $\mathbb{E}^L(n|y_L^*) = \frac{1}{F^L(y_L^*)} = 1.58$ and $k_H^* = 1.58$. Therefore, $k = \lceil k_H^* \rceil = 2$. Similarly, we have $\mathbb{E}^H(n|y_H^*) = \frac{1}{F^H(y_H^*)} = 4.47$ and $\mathbb{E}^H(n|y_L^*) = \frac{1}{F^H(y_L^*)} = 12.65$. An H-type agent refuses to search given the threshold y_L^*, because $\bar{V} - c\mathbb{E}^H(n|y_L^*) = -0.53$.

To check that the principal gets a higher utility proposing a stopping rule with threshold y_L^* than a stopping rule with a higher common threshold that allows H-type participation, notice that y^* can be derived from (5),and $y^* = 1.11$. Principal's expected utility by proposing a stopping rule with common threshold y^* is

$$\mathbb{E}U(y^*) = 0.7(1.26) + 0.3(0.006) = 0.88$$

while her expected utility using y_L^* is

$$\mathbb{E}U(y_L^*) = 0.7(1.367) = 0.957.$$

Now, consider the search rule $R^H = (2, y_H^*)$. First, an H-type agent participates, as $\bar{V} - c\mathbb{E}^H(n|R^H) = 1.06 > 0$. Second, the principal's expected utility from the

H-type agent is strictly positive, as $\bar{V} - c\mathbb{E}^H(n|R^H) - \mathbb{E}^H(p|R^H) = 0.18 > 0$. Therefore, the principal's expected utility offering the menu $\{y_L^*, (2, y_H^*)\}$ is

$$\mathbb{E}U(y_L^*, (2, y_H^*)) = 0.7(0.957) + 0.3(0.18) = 1.011 > \mathbb{E}U(y_L^*).$$

Now, we check that the sufficient condition of the proposition above holds. We have $u = \bar{V} - \mathbb{E}^H(p|y_H^*) - c(\mathbb{E}^H(n|y_H^*)) = 0.21$. Furthermore, by solving $f(z) = z + \dfrac{(1-x)^z}{x} - \dfrac{1}{x} = 1.05$ for z, we get $z = 3.82$. Therefore, if $k_H^* < 3.82$, then H-type participation is weakly beneficial by imposing the rule R^*. ♢

Another sufficient condition considers a kind of dual situation with respect to the previous one in which in the first best the probability that the H-type agent has to search more than k_H^* times is almost zero. It is clear that the overprovision of search effort by H-type is the cause of the inefficiency of the menu $R^* = \{R_L^*, R_H^*\} = \{(0, y_L^*), (k_H^*, y_H^*)\}$. If the benefit of H-type participation under the first best stopping rule is larger than the cost of overprovision of effort, then H-type agent's participation is profitable for the principal. Note that this sufficient condition occurs when in the first best the expected number of searches for the H-type is less than k_H^*.

Proposition 4.8 *Suppose $(1 - F^H(y_H^*))^{k_H^*}$ is close to zero and $\bar{V} - \mathbb{E}^H(p|y_H^*) > ck_H^*$. Then the principal gets a non-negative expected utility from the H-type agent proposing the search rule R_H^*.*

Proof. If $(1 - F^H(y_H^*))^{k_H^*}$ is close to zero, then

$$\mathbb{E}^H(n|R_H^*) = k_H^* + (1 - F^H(y_H^*))^{k_H^*}\, \mathbb{E}^H(n|y_H^*) \simeq k_H^*$$

and since by construction $E^H(p|R_H^*) \leq E^H(p|y_H^*))$, then $\bar{V} - E^H(p|y_H^*) > ck_H^*$, is a sufficient condition to guarantee that $H - type$ agent's participation is profitable for the principal. ■

The example below illustrates a situation in which this sufficient condition occurs.

Example 4.9 Let box $L = \{5, 10, 20\}$, with probability distribution $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ and $H = \{20, 100\}$ with probability distribution $(\frac{1}{100}, \frac{99}{100})$. Assume that the cost of search is $c = 2$. Finally let $V = 25$ $\bar{V} = 104$ and probability that agent is type $a \in \{H, L\}$ is $\frac{1}{2}$. It is easy to check that at the first best, $y^*_L = 10$ and $y^*_H = 100$, and if the principal proposes a threshold strictly lower than 100, then H-type agents do not participate, because $104 - 100(2) < 0$. Under the first best rule the expected number of draws for the L-type is 1.5 and her expected utility is $25 - 1.5(2) = 22$; the expected utility of the principal when only L-type agent accepts the contract is $\frac{1}{2}(25 - 1.5(2) - 7.5). = 7.25$. If the principal proposes a stopping rule with common threshold equal to 100 she gets

$$\frac{1}{2}(25 - \frac{35}{3} - 2) + \frac{1}{2}(104 - (\frac{20}{100} + 99) - 2) \simeq 7.07.$$

Hence, it is more profitable for the principal to propose a search rule with threshold $y^*_L = 10$ than one with threshold $y^* = 100$. One can derive $k^*_H = 1.5$ and $k = \lceil k^*_H \rceil = 2$. If the principal proposes the search rule $R_H = (2, 100))$ to the H-type agent, the H-type agent is going to stop with probability one after two searches (and therefore she accepts to search under this rule). The probability that the minimum of the two draws is 20 is $\frac{1}{100}.\frac{1}{100} + 2(\frac{1}{100}.\frac{99}{100}) = 0.0199$ and the probability that the minimum of the two draws is 100 is $\frac{99}{100}.\frac{99}{100} = 0.9801$. Therefore,

$$\mathbb{E}^H(p|R_H) = 20(0.0199) + 100(0.9801) = 98.408,$$

Finally, the expected utility of the principal offering the IC menu $\{(0, 10), (2, 100)\}$ is

$$\frac{1}{2}(25 - 1.5(2) - 7.5) + \frac{1}{2}(104 - 4 - 98.408) = 8.046.$$

which is higher than the expected utility of the principal when she offers the search rule with common threshold $y^*_L = 10$ and only L-type agents accept to search. Notice that $\bar{V} - \mathbb{E}^H(p \mid y^*_H) = 104 - 98.408 = 5.592 > 4 = ck$. $\diamondsuit$

Now, we discuss the case in which $y^*_L \geq y^m_H$. According to Proposition 4.4, the optimal common threshold for the stopping rule in this case is $y^* \in (y^*_L, y^*_H)$. We have the following result:

Proposition 4.10 *Assume $y_L^* \geq y_H^m$. If $k_H^* + \mathbb{E}^H(n|y_H^*) \leq \mathbb{E}^H(n|y^*)$, then principal gets higher payoff offering the menu of IC rules R^* than proposing a stopping rule with common threshold y^*.*

Proof. As we assume $y_L^* \geq y_H^m$, according to Proposition 4.4, the optimal common threshold for the stopping rule is $y^* \in (y_L^*, y_H^*)$. Then, the principal's expected utility is

$$\mathbb{E}U^P(y^*) = \rho U_L^P(y^*) + (1-\rho)U_H^P(y^*).$$

Moreover, the principal's expected utility given the menu R^* is

$$\mathbb{E}U^P(R^*) = \rho U_L^P(y_L^*) + (1-\rho)U_H^P(R_H^*).$$

Clearly, $U_L^P(y_L^*) \geq U_L^P(y^*)$. So, it is enough to show that if

$$k_H^* + \mathbb{E}^H(n|y_H^*) \leq \mathbb{E}^H(n|y^*),$$

then $U_H^P(R_H^*) \geq U_H^P(y^*)$. We also know that the expected price under R_H^* is less than the expected price in the first best. Then, we have

$$\begin{aligned} U_H^P(R_H^*) &= \bar{V} - c\mathbb{E}(n|R_H^*) - \mathbb{E}(p|R_H^*) \\ &\geq \bar{V} - c\mathbb{E}(n|R_H^*) - \mathbb{E}(p|y_H^*). \end{aligned}$$

Now, if $\mathbb{E}(n|R_H^*) \leq \mathbb{E}^H(n|y^*)$, then the principal weakly prefers the menu R^* to the stopping rule with the optimal common threshold y^*. Furthermore, we have

$$\mathbb{E}(n|R_H^*) = k_H^* + (1 - F^H(y_H^*))^{k_H^*}\mathbb{E}^H(n|y_H^*) < k_H^* + \mathbb{E}^H(n|y_H^*).$$

This completes the proof. ∎

5 Discussion

This paper provides a theoretical explanation to the practice of imposing a minimum number of searches to agents who are delegated to make a choice on behalf of the principal. Delegation opens the door to problems of moral hazard and adverse selection, which are especially severe when the principal cannot offer monetary incentives to the agent. In our model, a benevolent principal delegates an agent to buy the good she needs, and pays its price. Since agents do not pay the good but the search cost, they do not have incentive to spend enough effort in searching for the best price (moral hazard problem). Moreover, since different types of agents need different quality of goods, and prices are correlated with quality, a principal, even if she knows the price distribution for each quality of goods and the search cost, cannot impose to the agent to search optimally (adverse selection).

We restrict our attention to the simple class of stopping rules with a minimum number of searches and a thresholds. The requirement of a minimum number of searches helps the principal to restore the incentive compatibility at the cost of inducing an overprovision of effort to the agents. We show that under some conditions it is optimal for the principal to propose a menu of incentive compatible search rules that imposes to a high type agent a minimum number of searches. Namely, a low type agent can stop searching when she finds a price lower than her first best threshold, while the high type agent can stop search if she finds a price lower than her first best threshold, *and* she has performed a minimum number of searches.

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